

## 1 Hashing

(a) Imagine we have the following class:

```
1 public class Course {
2     public final int CCN;
3     public final String instructor;
4     public Student[] students;
5     public int audited; //when the course was last audited
6     public Course(int CCN, Student[] initial) {
7         this.CCN = CCN;
8         this.students = initial;
9         this.instructor = "Sohum";
10    }
11    //implementation
12    public void audit() {
13        this.audited = System.currentTimeMillis();
14        //implementation
15    }
16    public void addStudent(Student s) {
17        //implementation
18    }
19 }
```

Which of the following hashing functions for the Course class are **valid**?

A)

```
1 @Override
2 public int hashCode() {
3     return CCN;
4 }
```

B)

```
1 @Override
2 public int hashCode() {
3     return this.students.length;
4 }
```

C)

```
1 @Override
2 public int hashCode() {
3     return this.audited;
4 }
```

2 Hashing

D)

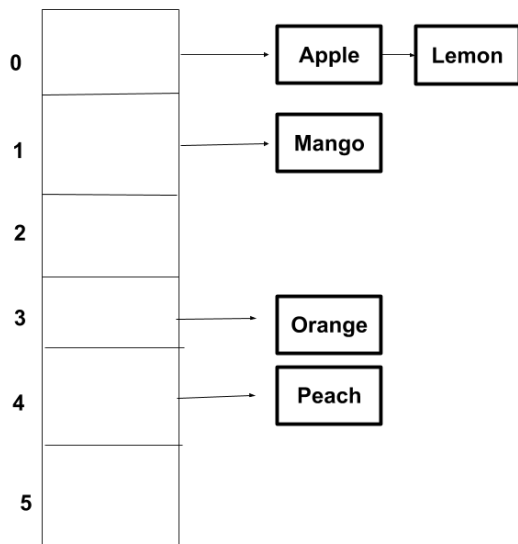
```
1 @Override
2 public int hashCode() {
3     return 5;
4 }
```

E)

```
1 @Override
2 public int hashCode() {
3     return getNumericValue(this.instructor.charAt(0));
4 }
```

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E

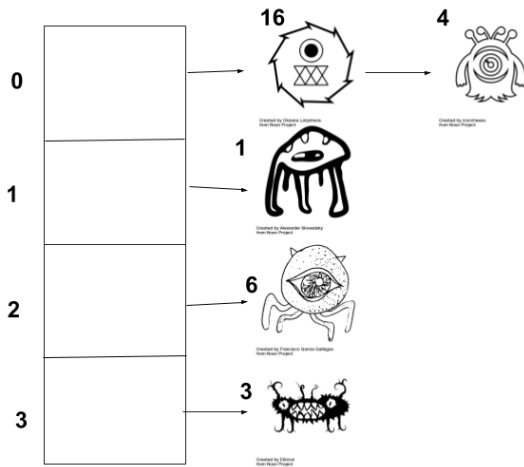
(b) We have the below external chaining HashSet.



If the load factor is 1.25, how many more insertions can we make before we will resize? **Do not** include the insertion that will begin with the resize.

- 1
- 2
- 3
- 4
- 5
- 6

(c) Suppose we have the following MonsterHashTable.



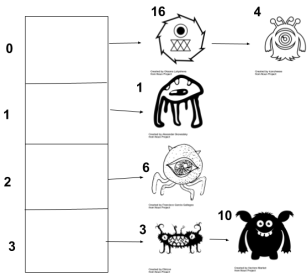
The number to the upper left of the monster is their hashCode.

**Part One:**

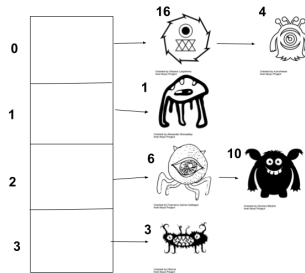
Suppose we want to insert the element:



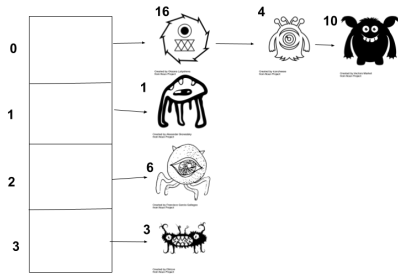
Which of the following would correctly mirror the state after inserting the above element?



A)



B)



C)

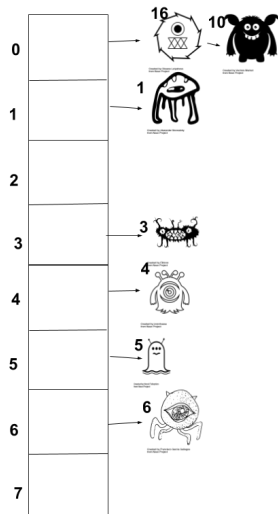
- A
- B
- C

**Part Two:**

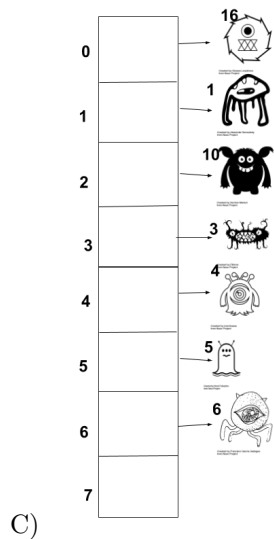
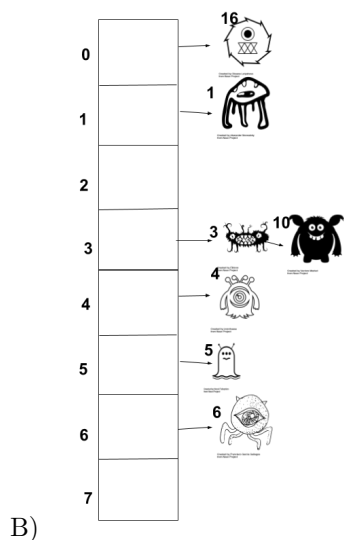
Now, after inserting that element, we want to insert a new element (shown below) **and** resize.



Which of the following correctly mirrors the new state?



A)



- A
- B
- C

## 2 Unexpected Hashing

Suppose we have the Lamp class below:

```

1 class Lamp {
2     int brightness;
3
4     Lamp(int brightness) {
5         this.brightness = brightness;
6     }
7
8     @Override
9     public int hashCode() {
10        return brightness;
11    }
12
13    @Override
14    public boolean equals(Object o) {
15        return ((Lamp) o).brightness == brightness;
16    }
17 }

```

Assume the HashMap is implemented with external chaining. Assume the size of the internal array of the HashMap is 2 and doesn't resize. Determine the output of each print line below:

```

1 Lamp a = new Lamp(1);
2 Lamp b = new Lamp(2);
3
4 HashMap<Lamp, Integer> map = new HashMap<>();
5
6 map.put(b, 0);
7 map.put(a, 1);
8 map.put(a, 2);
9
10 System.out.println(map.get(a)); // print statement 1 2
11 System.out.println(map.get(b)); // print statement 2 0
12
13 map.put(b, 3);
14 a.brightness = 2;
15 map.put(b, 4);
16
17 System.out.println(map.get(a)); // print statement 3 4
18 System.out.println(map.get(b)); // print statement 4 4
19 System.out.println(map.get(new Lamp(1))); // print statement 5 null

```

Lamp  
a  
2

Lamp  
b  
2

→ 0 → (b, 4)  
1 → (a, 2)

↖ a → 2  
↗ 2 equals

↖ hc: 2 → i: 0

Lamp  
1

**Print Statement 1:**

- 0
- 1
- 2
- 3
- 4
- null

**Print Statement 2:**

- 0
- 1
- 2
- 3
- 4
- null

**Print Statement 3:**

- 0
- 1
- 2
- 3
- 4
- null

**Print Statement 4:**

- 0
- 1
- 2
- 3
- 4
- null

**Print Statement 5:**

- 0
- 1
- 2
- 3
- 4
- null

Login: \_\_\_\_\_

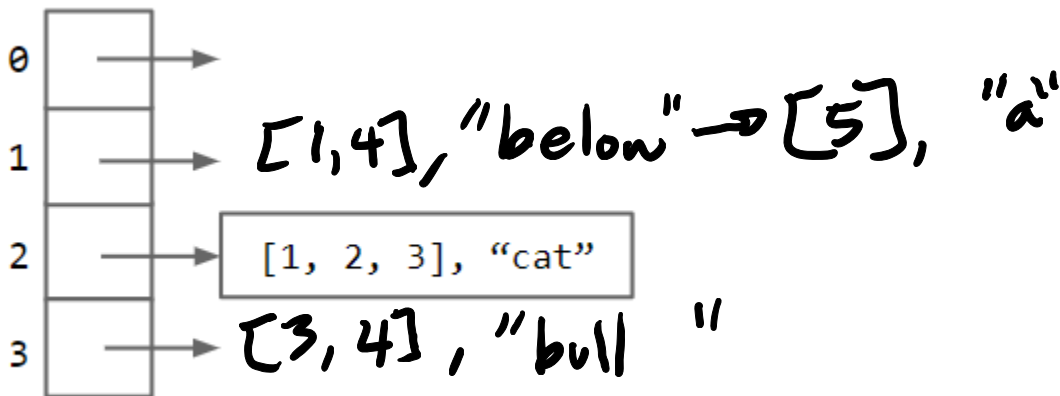
e) (3 points). Draw a valid BST of minimum height containing the keys 1, 2, 3, 7, 8, 9, 5.

f) (6 points). Under what conditions is a **complete** BST containing N items **unique**? By unique we mean the BST is the only complete BST that contains exactly those N items. By complete we mean the same idea that was required for a tree to be considered a heap (not repeated here). Reminder: We never allow duplicates in a BST.

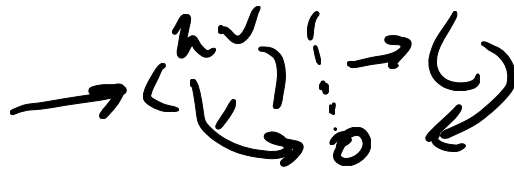
**2. Hash Tables.**

a) (5 points). Draw the hash table that is created by the following code. Assume that `XList` is a list of integers, and the hash code of an `XList` is the sum of the digits in the list. Assume that `XLists` are considered equal only if they have the same length and the same values in the same order. Assume that `FourBucketHashMaps` use external chaining and that new items are added to the end of each bucket. Assume `FourBucketHashMaps` always have four buckets and never resize. The result of the first `put` is provided for you. Represent lists with square bracket notation as in the example given.

```
FourBucketHashMap<XList, String> fbhm = new FourBucketHashMap<>();
fbhm.put(XList.of(1, 2, 3), "cat");
fbhm.put(XList.of(1, 4), "riding");
fbhm.put(XList.of(5), "a");
fbhm.put(XList.of(3, 4), "bull");
fbhm.put(XList.of(1, 4), "below");
```







b) (4.5 points). Next to the calls to get, write the return value of the get call. Assume that get returns null if the item cannot be found.

```
FourBucketHashMap<XList, String> fbhm = new FourBucketHashMap<>();
```

```
XList firstList = XList.of(1, 2, 3);
```

```
fbhm.put(firstList, "cat");
```

```
fbhm.get(XList.of(1, 2, 3));
```

```
firstList.addLast(0); // list is now [1, 2, 3, 0]
```

```
fbhm.get(firstList);
```

```
fbhm.get(XList.of(1, 2, 3));
```

cat  
 cat  
 null  
 → right hc, wrong =

no change to hc

c) (10.5 points). Next to the calls to get, write the return value(s) of the get call. Assume that get returns null if the item cannot be found.

```
FourBucketHashMap<XList, String> fbhm = new FourBucketHashMap<>();
```

```
XList firstList = XList.of(1, 2, 3);
```

```
fbhm.put(firstList, "cat");
```

```
firstList.addLast(1); // list is now [1, 2, 3, 1]
```

```
fbhm.get(firstList);
```

```
fbhm.get(XList.of(1, 2, 3));
```

```
fbhm.get(XList.of(1, 2, 3, 1));
```

```
fbhm.get(XList.of(3, 4));
```

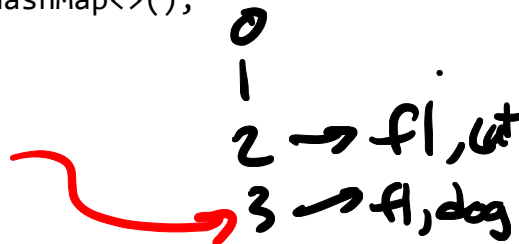
```
fbhm.put(firstList, "dog");
```

```
fbhm.get(firstList);
```

```
fbhm.get(XList.of(1, 2, 3));
```

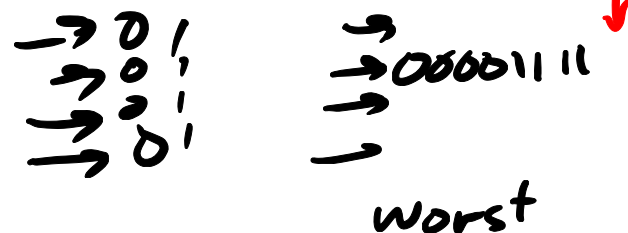
```
fbhm.get(XList.of(1, 2, 3, 1));
```

lost access  
 null  
 null  
 null  
 null  
 dog  
 null  
 dog



d) (4 points). What are the best and worst case get and put runtimes for FourBucketHashMap as a function of N, the number of items in the HashMap? Don't assume anything about the distribution of keys.

.get best case:  $\Theta(1)$   
 .get worst case:  $\Theta(N)$   
 .put best case:  $\Theta(1)$   
 .put worst case:  $\Theta(N)$



e) (4 points). If we modify FourBucketHashMap so that it triples the number of buckets when the load factor exceeds 0.7 instead of always having four buckets, what are the best and worst case runtimes in terms of N? Don't assume anything about the distribution of keys.

.get best case:  $\Theta(1)$   
 .get worst case:  $\Theta(N)$   
 .put best case:  $\Theta(1)$   
 .put worst case:  $\Theta(N)$

As noted on the front page, throughout the exam you should assume that a single resize operation on any hash map takes linear time.