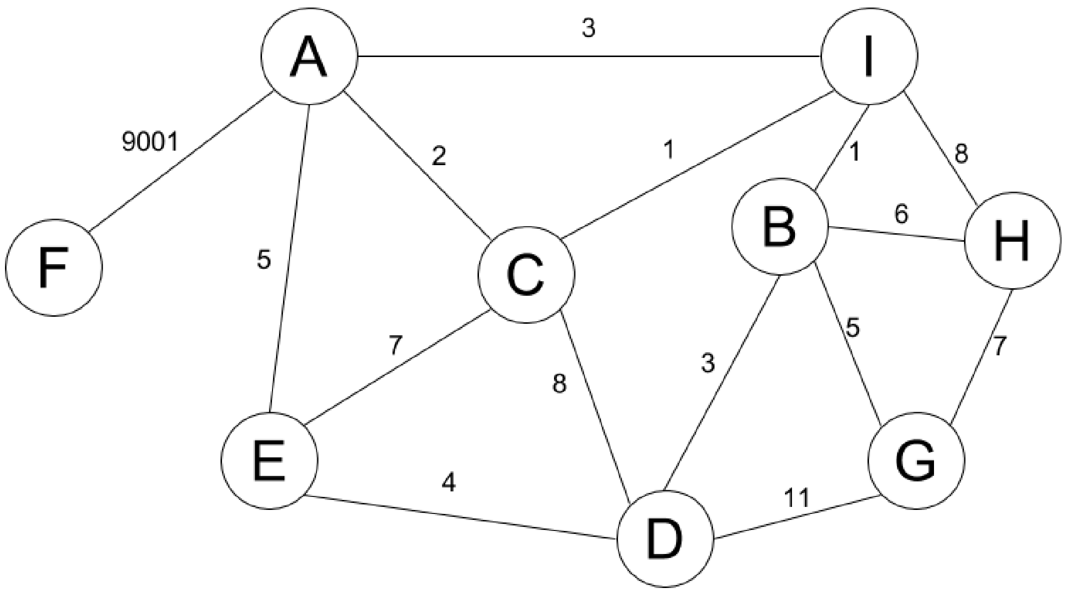
CS 61BL Summer 2021 | Shortest Paths and MSTs
Quiz 11: Wednesday August 4, 2021

1 Prims

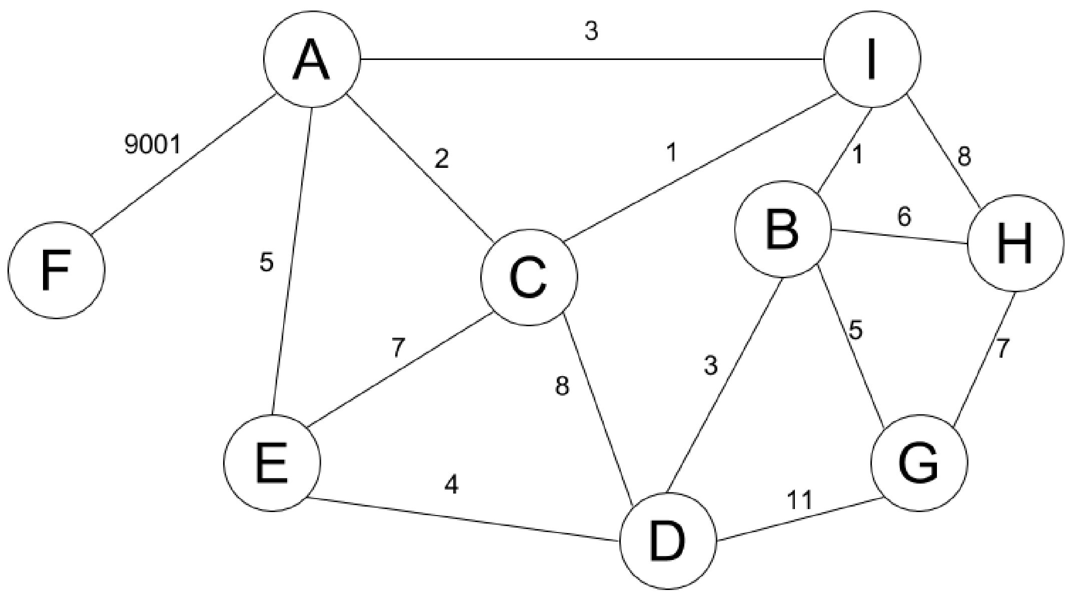
Suppose we have the graph below, and run Prim's algorithm on the graph **starting from vertex A**.



What is the order that vertices are visited? Enter your answer as a space separated list, e.g. `A B C`.

2 Kruskals

Suppose we have the same graph from before, which has been recopied below. Run Kruskal's algorithm on the below graph.



(a) Select the edges that are included in the resulting MST. Note, as this is an undirected graph the order of the elements in the edge does not matter. In other words, edge A-B is exactly equivalent to edge B-A. For simplicity, we will only include the alphabetically ordered edges.

[ ] A-F [ ] A-C [ ] A-E [ ] A-I [ ] C-E [ ] C-D [ ] C-I [ ] D-E [ ] D-G [ ] B-I [ ] B-D [ ] B-G [ ] B-H [ ] H-I [ ] G-H

(b) If we change the edge `AF` to 0, would this change the MST found by Kruskals'?

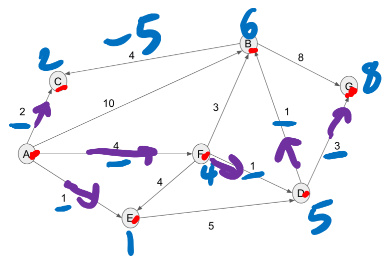
( ) Yes
( ) No

(c) If we change the edge `DG` to 4, would this change the MST found by Kruskals'?

( ) Yes
( ) No

3 Dijkstra's

All parts of this question refer to the below graph.



Recall Dijkstra's shortest paths algorithm, which finds the shortest path from a starting vertex `v` to all other vertices in the graph. Run Dijkstra's algorithm on the above graph **starting at vertex A**.

(a) What is the order that vertices are visited? Enter your answer as a space separated list, e.g. `A B C`.

A E C F D B G

(b) What is the final `edgeTo` map?

( ) {`'A': null, 'B': 'F', 'C': 'A', 'D': 'F', 'E': 'D', 'F': 'A', 'G': 'D'`}
( ) {`'A': null, 'B': 'A', 'C': 'A', 'D': 'E', 'E': 'A', 'F': 'A', 'G': 'D'`}
( ) {`'A': null, 'B': 'F', 'C': 'A', 'D': 'F', 'E': 'A', 'F': 'A', 'G': 'D'`}
( ) {`'A': null, 'B': 'D', 'C': 'A', 'D': 'E', 'E': 'A', 'F': 'A', 'G': 'D'`}
(X) {`'A': null, 'B': 'D', 'C': 'A', 'D': 'F', 'E': 'A', 'F': 'A', 'G': 'D'`}
( ) None of the above

(c) What is the maximum value we can change the edge `BC` to so that running Dijkstra's on the modified graph from A **fails to the shortest path** to C? Note that this is a challenging problem so we recommend only attempting it after finishing the rest of the quiz. Hint: Make the edge negative.

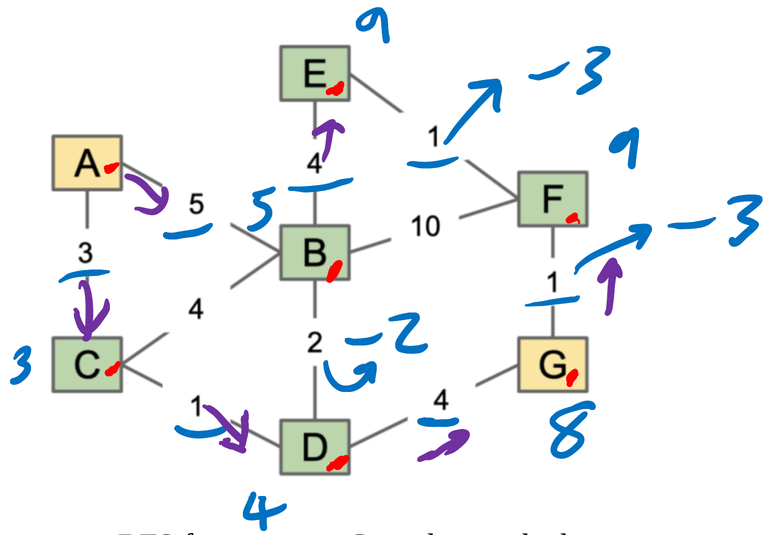
Dis: 1 Curr: 2

$4+1+1+BC=1$

$BC=-5$

9. Shortest Paths (220 points).

Consider the graph below. We recommend you draw the graph on paper before starting. For all algorithms, suppose we break ties alphabetically.



a) **BFS (30 points).** Suppose we run BFS from **vertex G** on the graph above. Give the first **four** vertices visited by BFS, including the start **vertex G**. Format your answer as a comma separated list, e.g. "G, X, Y, Z".

Order: ____

b) **Dijsktras (140 points).** Suppose we run Dijsktra's from **vertex A** on the graph above. Note that we do not ask for distTo(A) because it's trivially zero since A is the start.

1. (25 points). Give the distance values computed by Dijkstra's algorithm for each vertex, if we run Dijkstra's starting at **vertex A**.

   `distTo(B):` 5　　`distTo(C):` 3　　`distTo(D):` 4

   `distTo(E):` 9　　`distTo(F):` 9　　`distTo(G):` 8

2. (25 points). What is the order that the vertices are visited by Dijkstra's algorithm starting from vertex A? Format your answer as a comma separated list, including vertex A.

   Order: A C D B G E F

3. (25 points). What edges are included in the shortest paths tree (SPT) for vertex A?

   ☒ AC　☒ AB　☐ BC　☐ BD (BE)　☐ BF　☒ CD　☒ DG　☐ EF　☒ FG

4. (65 points). **Subtract an integer k** from **one edge** such that Dijkstra's fails to find the shortest path from **A to G**. For instance, if we wanted to change BE to 1, **k** would be 3. What is the **minimum k**, and what single edge should be changed? If there are multiple correct answers check all that apply (e.g. if subtracting k from AC by itself or subtracting k from AB by itself would result in the wrong shortest path from **A to G**, check the boxes for both AC and AB).

To reemphasize: We're only concerned with the correctness of the shortest path from **A to G**.

☐ AC ☐ AB ☐ BC ☑ BD ☐ BF ☐ CD ☐ DG ☑ EF ☑ FG

k: 42.

c) **A* (65 points).** Suppose we run A* from **A** to **G** on the original graph with the heuristic function below. Note that we are trying to find the shortest path from **A to G**, *but* the heuristic of a vertex **v** is the weight of the shortest path from **v to E.**

```
h(v) = total weight of the shortest path from v to E
```

For example, `h(E) = 0, h(F) = 1, h(G) = 2, h(B) = 4`, etc.

1. What will be the priority of C when it is added to the fringe?

   Priority of C: ____________________________________

2. In what order will the vertices be visited? Give your answer as a comma separated list:

   A* visit order: ____

3. Will A* compute the correct shortest total distance from **A** to **G** using this heuristic?

   ○ Yes ○ Depends on how ties are broken by the priority queue ○ No