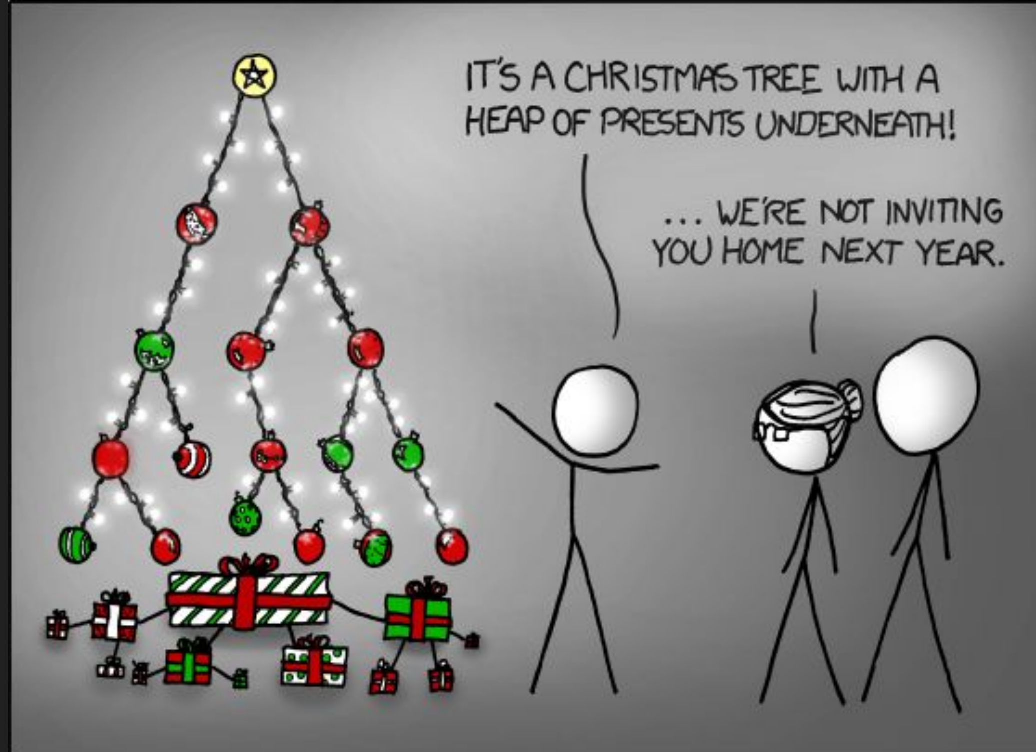
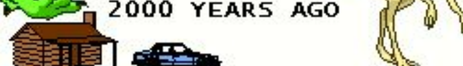
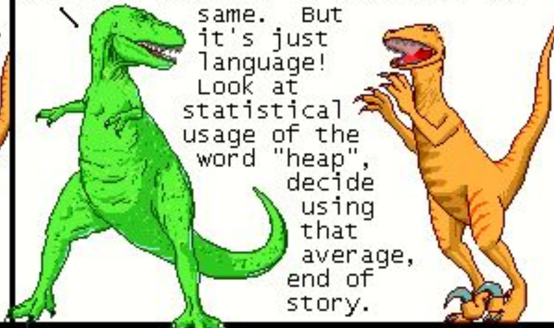
Lab 9: Priority Queues and Heaps



Announcements

- Lab 9 - due Friday 03/18
- HW 6 - due Tuesday 03/29
- Project 2: Ataxx
  - Checkpoint - due Friday 03/18
  - Project - due Friday 04/01





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Theory - Priority Queues

`Queue` Interface

Based on *time* (recency)

`enqueue(val)`	Adds `val` to the queue
`peek() / poll()`	Returns the item in the queue that was enqueued ***longest ago***.

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`PriorityQueue`: A `Queue` that prioritizes certain items (e.g hospital ER)

Examples:

`Queue` Interface

Based on *time* (recency)

`enqueue(val)`	Adds `val` to the queue
`peek() / poll()`	Returns the item in the queue that was enqueued ***longest ago***.

`PriorityQueue`: A `Queue` that prioritizes certain items (e.g hospital ER)

Examples:

- OS Process Scheduling
- Sorting
- Greedy algorithms (e.g. “shortest path”)

`PriorityQueue` ADT

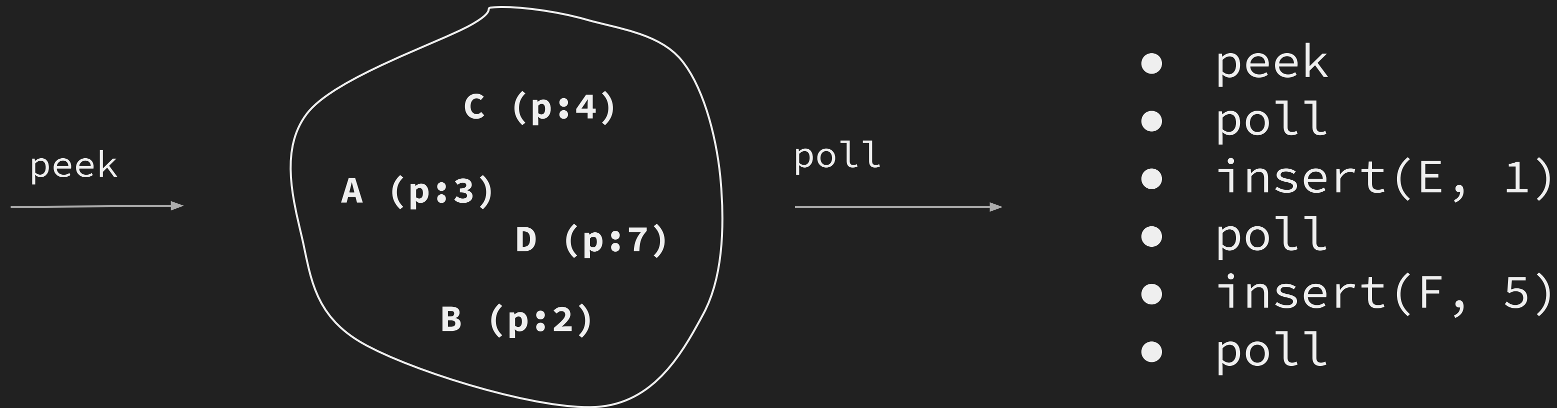
`insert(val, priority)`	Adds `val` to the queue with ***priority value*** `priority`
`peek() / poll()`	Returns the item in the queue with the ***highest priority***.

- Min priority queue: highest priority == *lowest priority value*
  - There's also a "max priority queue" / max heap, where highest priority value is highest priority
- No specification on how to deal with ties

`PriorityQueue` ADT

`insert(val, priority)`	Adds `val` to the queue with *priority value* `priority`
`peek() / poll()`	Returns the item in the queue with the ***highest priority***.

- Highest Priority == Lowest Priority Value



- peek
- poll
- insert(E, 1)
- poll
- insert(F, 5)
- poll

PriorityQueue ADT

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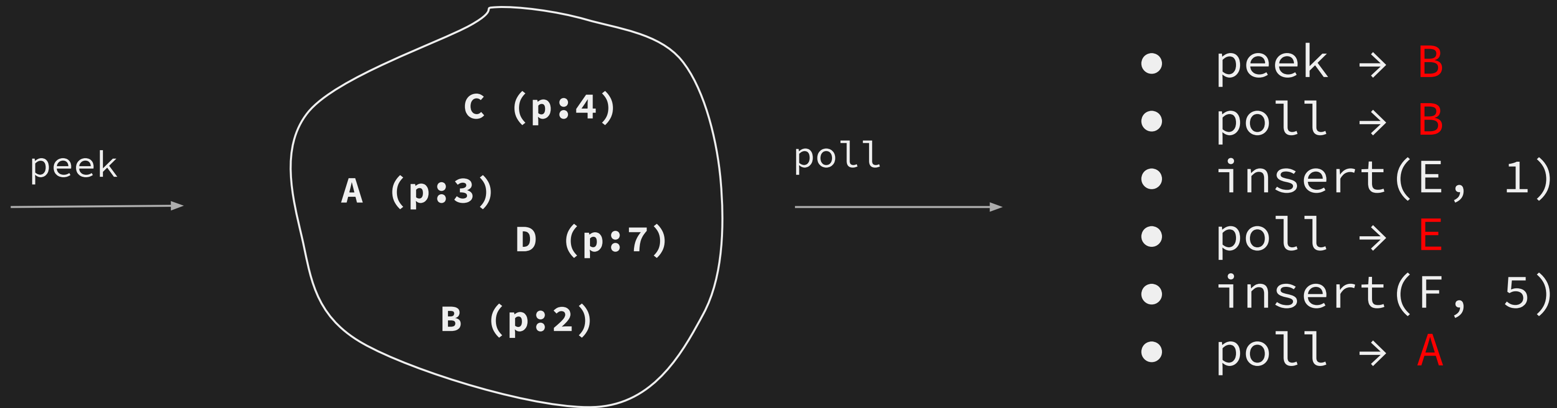


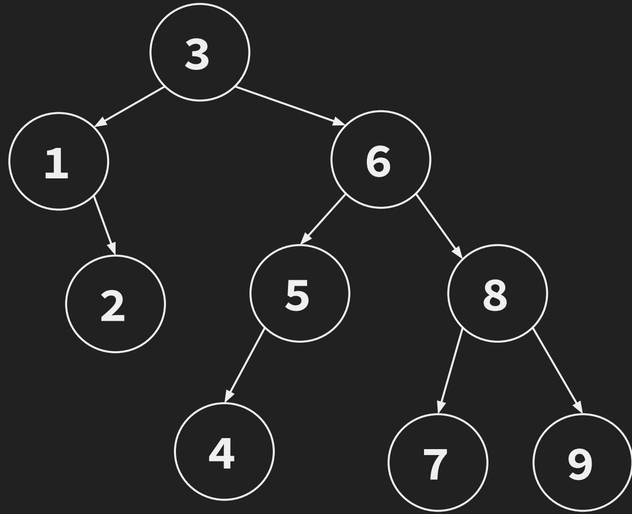
- peek → B
- poll → B
- insert(E, 1)
- poll → E
- insert(F, 5)
- poll → A

Theory - Heaps

Review: BST Properties

BST Property (recursive invariant)

- Left Children are smaller
- Right Children are larger



In Contrast: Heap Properties

BST Property (recursive invariant)
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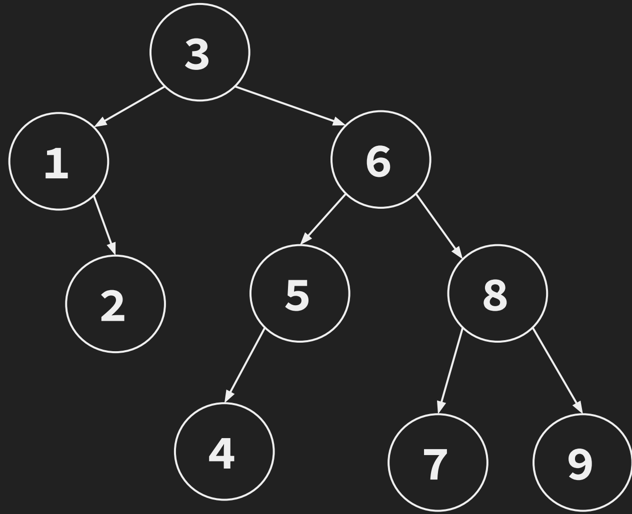
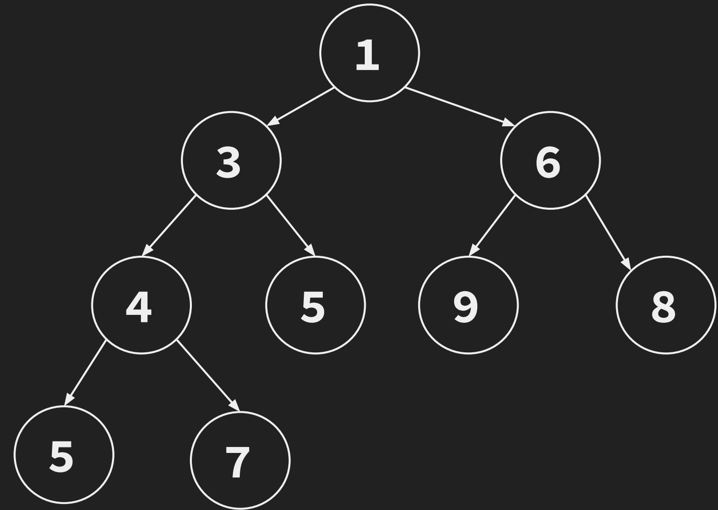
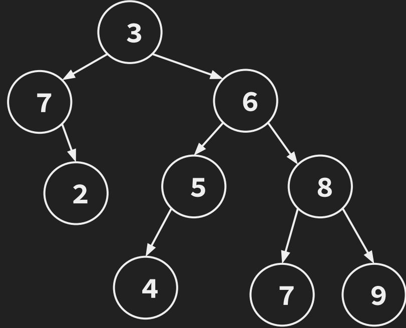


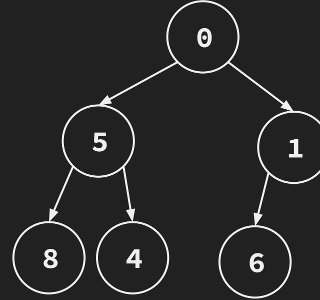
Min-Heap Property (recursive invariants)
- All Children are larger

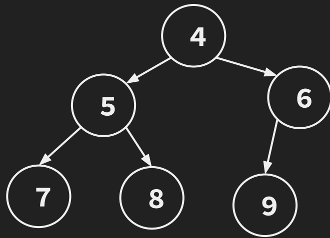
Completeness Property
- Tree has no “gaps” (left-packed)

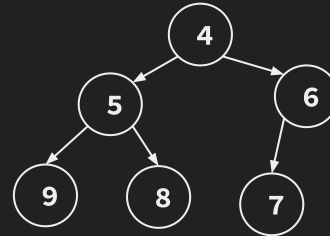


Does it Heap?

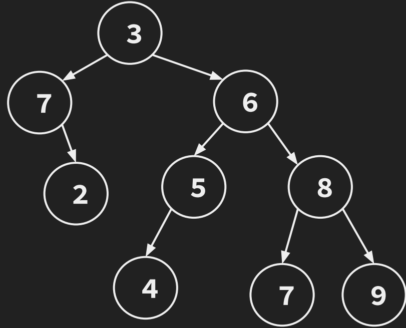




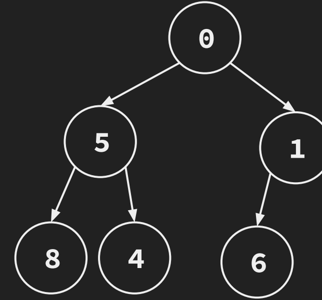




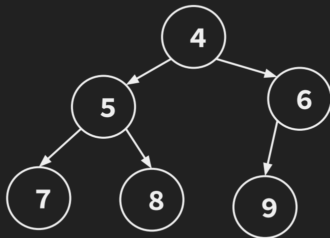
Does it Heap?



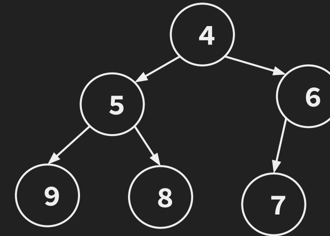
No - fails both invariants



No, fails min-heap invariant.

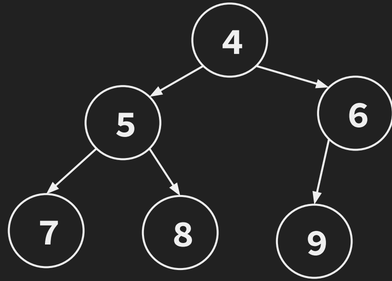


Yes!



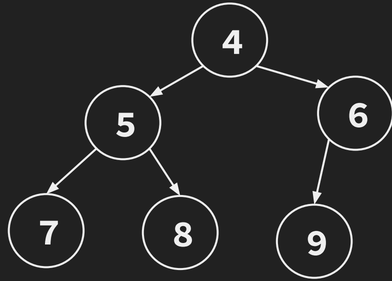
Yes!

Heap Properties



- `peek` - where's the element with the smallest priority?

Heap Properties



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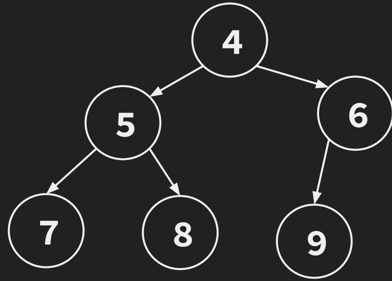
Heap Properties



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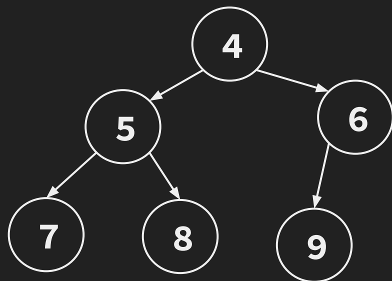
- Heap height with N items?

Heap Properties



- `peek` - where's the element with the smallest priority?

  Top of the heap!

- Heap height with N items?

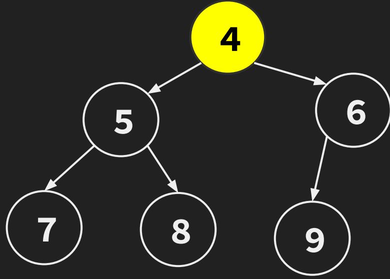
  Count total # of elements in full heap with K layers
  - # of elements in each layer doubles

  $N \approx 2^0+2^1+...+2^{K-1} = 2^K-1.$
  $\lg N \approx \lg 2^K-1 \approx \lg 2^K = K$

Theory-Implementation:
`removeMin`

removeMin

Find min



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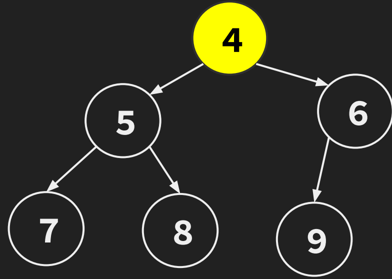
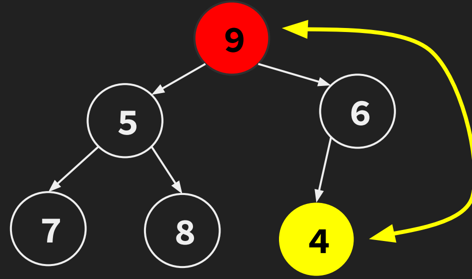


Swap with last child



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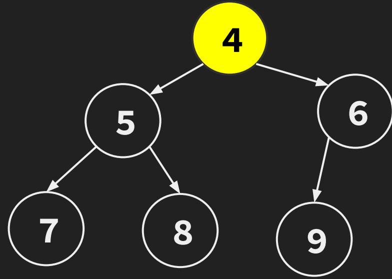

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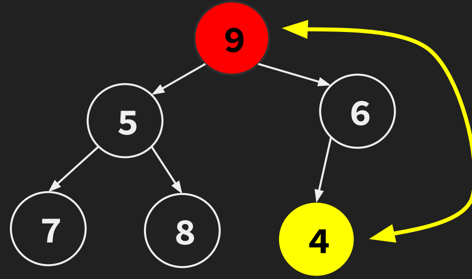
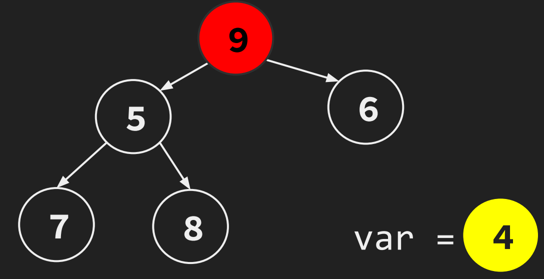


Delete and save previous min



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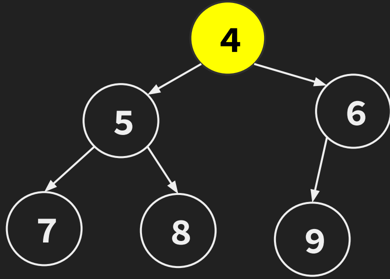

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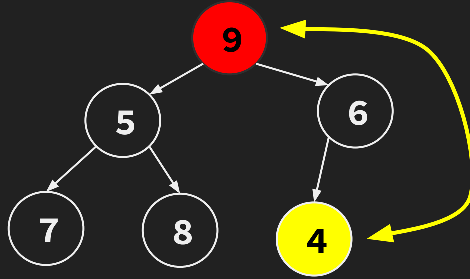

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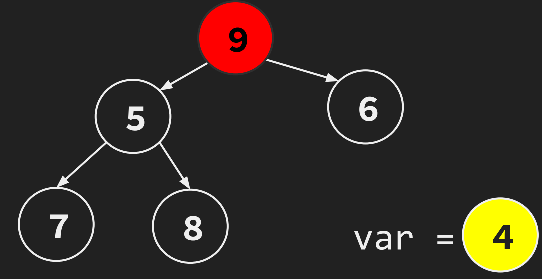
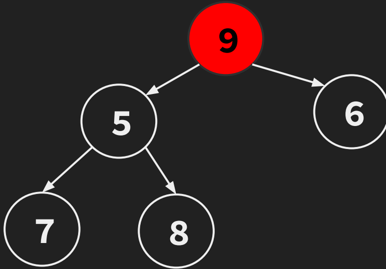


"Bubble Down" to fix heap invariant



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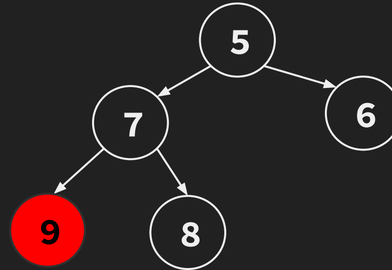

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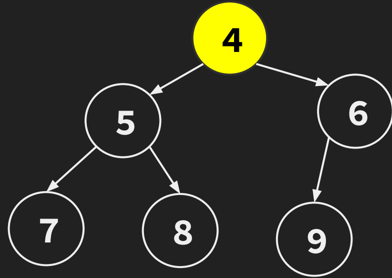

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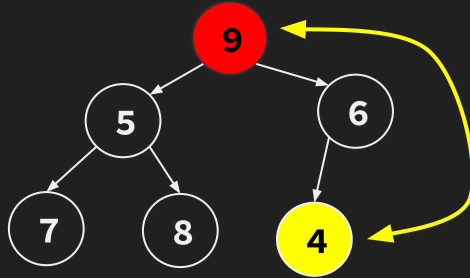

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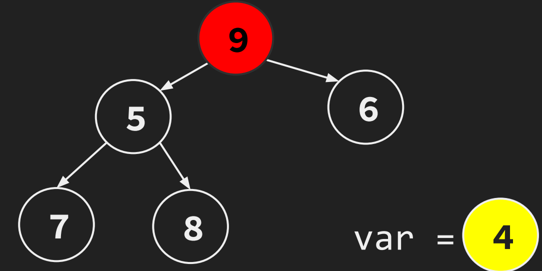

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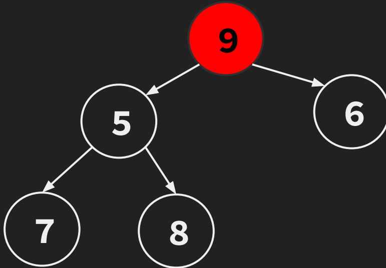

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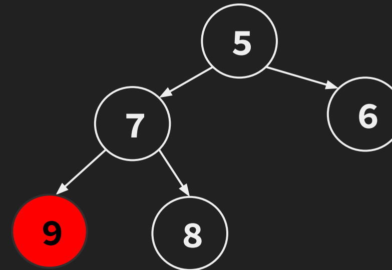

Return min

return 4

“Bubble Down”

```
bubbleDown(node) {
    while (node.priority is greater than either child) {
        Swap data with smaller child
    }
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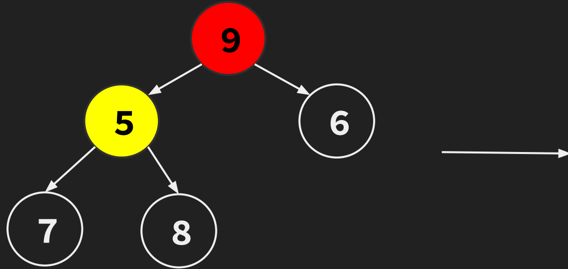
```



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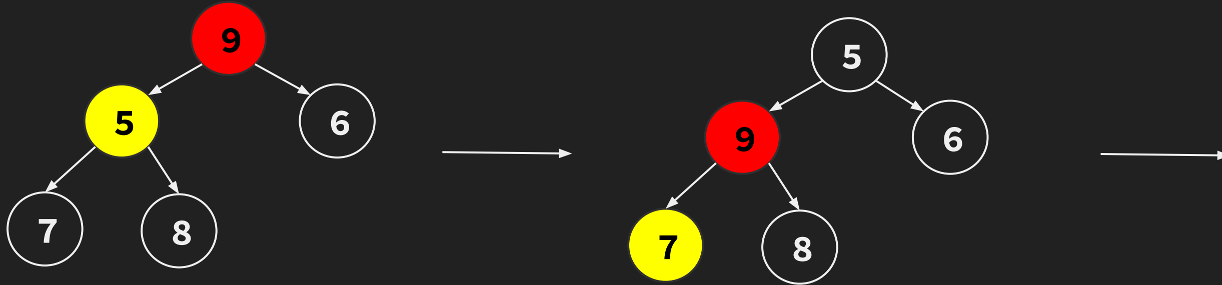

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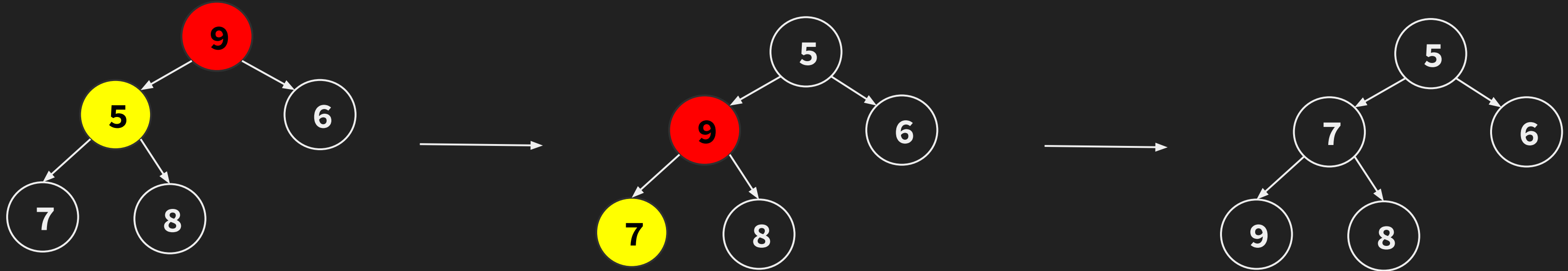

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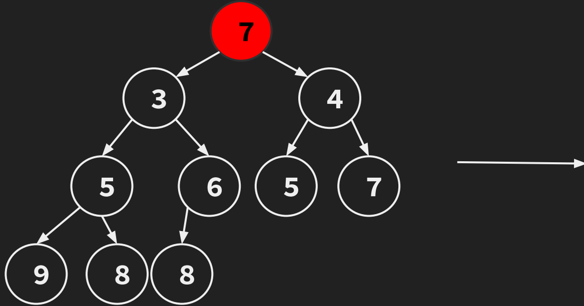

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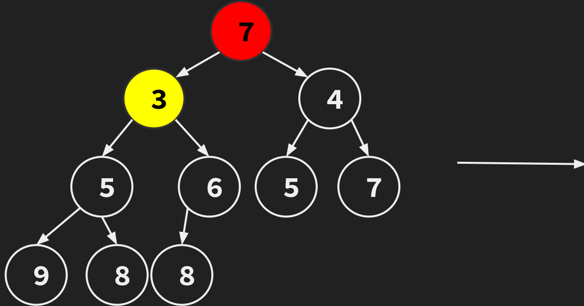

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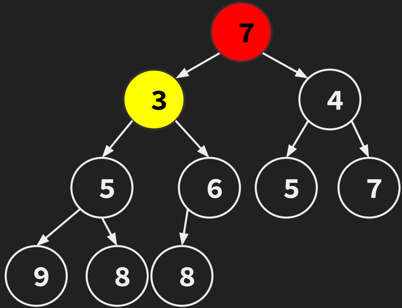

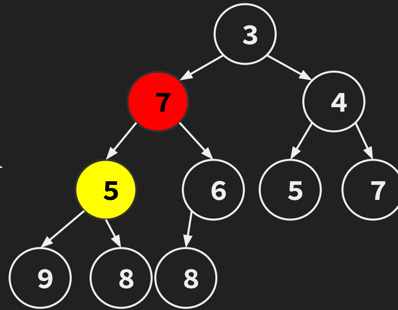

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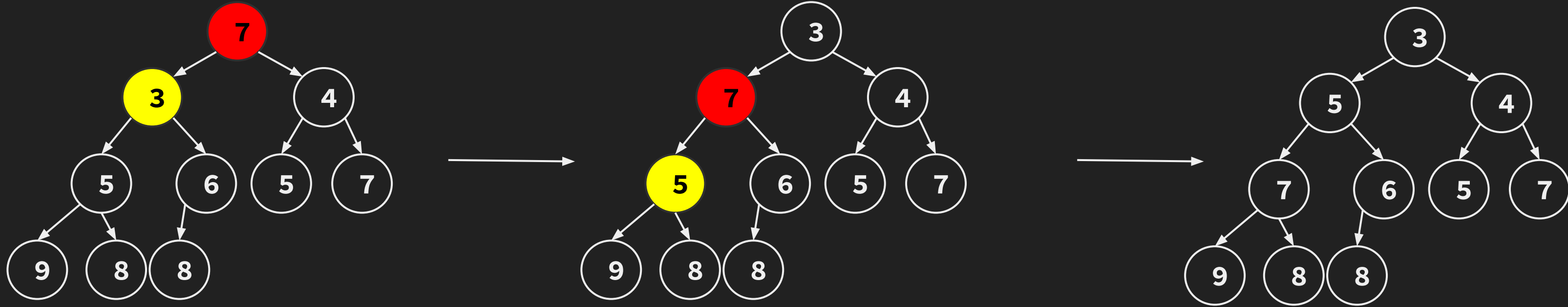

Runtime?

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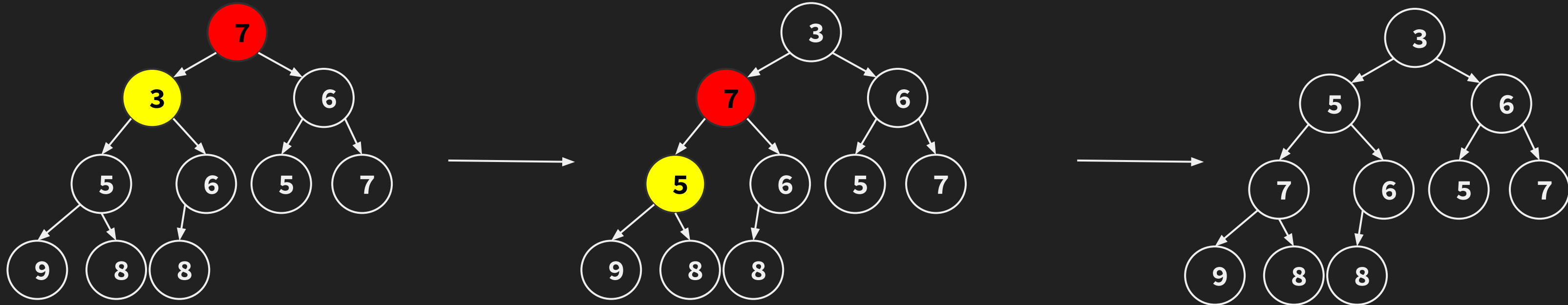
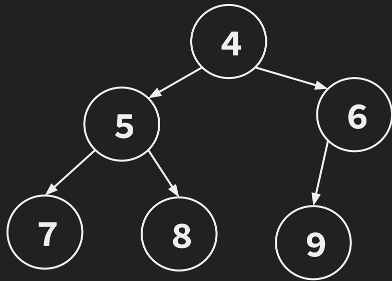


Runtime? Worst case, swap through every layer. Heap height is ~lg n, so runtime is O(lg n).

Theory-Implementation: `insert`

insert

`insert(1).`
Where should the new item go first?



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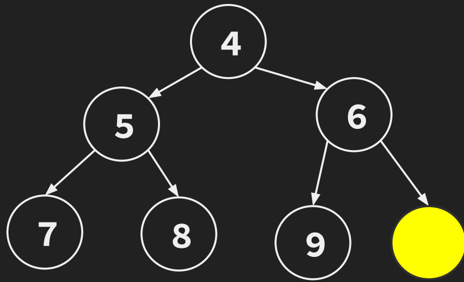

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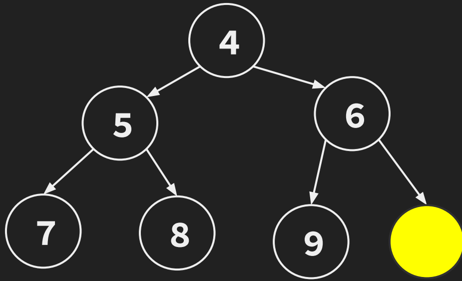
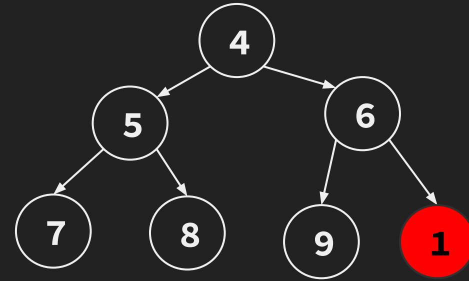


Fill last “hole” with 1.



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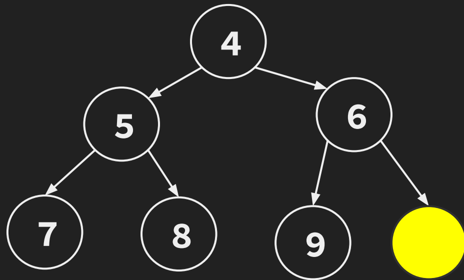

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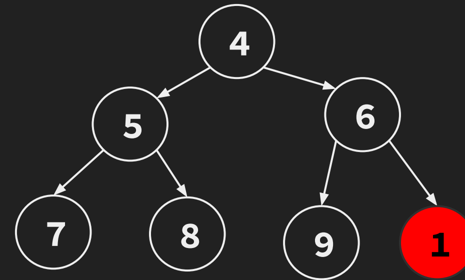
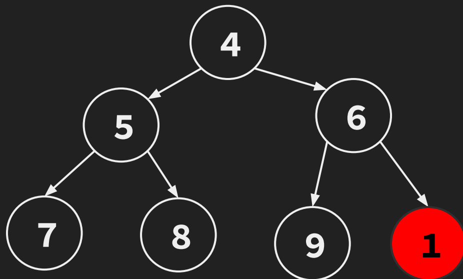


“Bubble Up” to fix heap invariant



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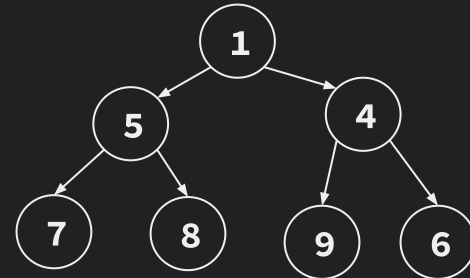

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    while (node.priority is smaller than parent) {
        Swap data with parent
    }
}
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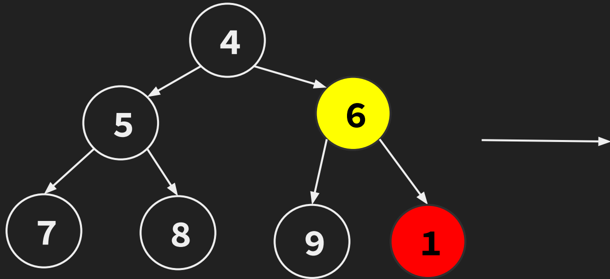

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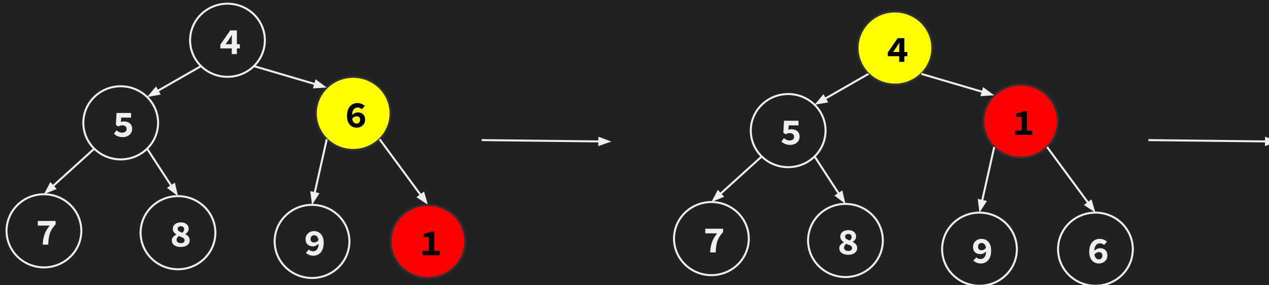

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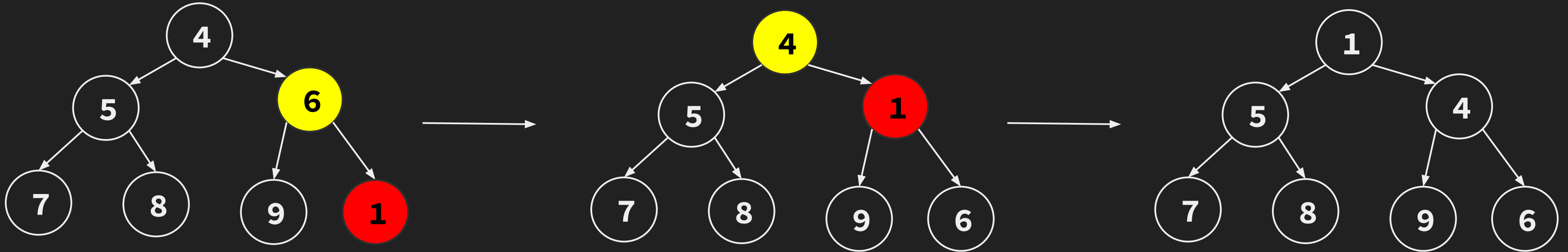

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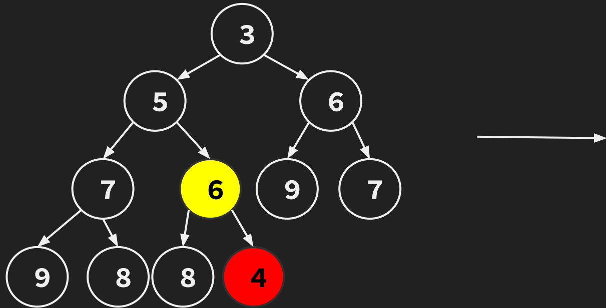

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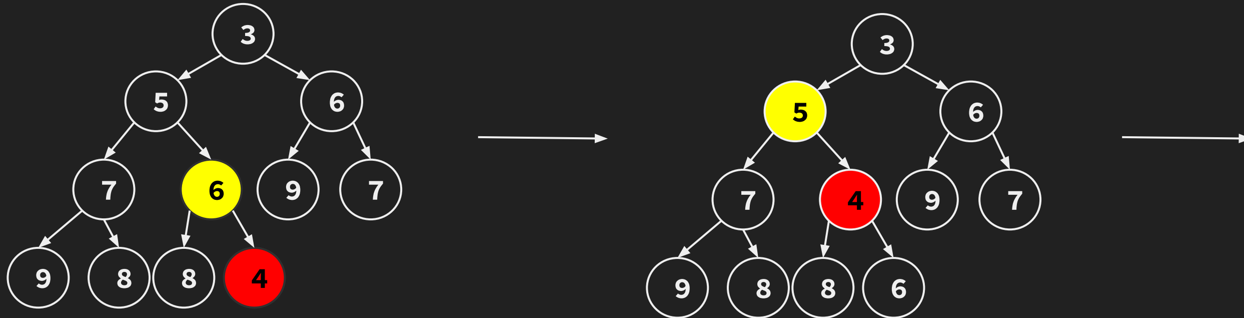

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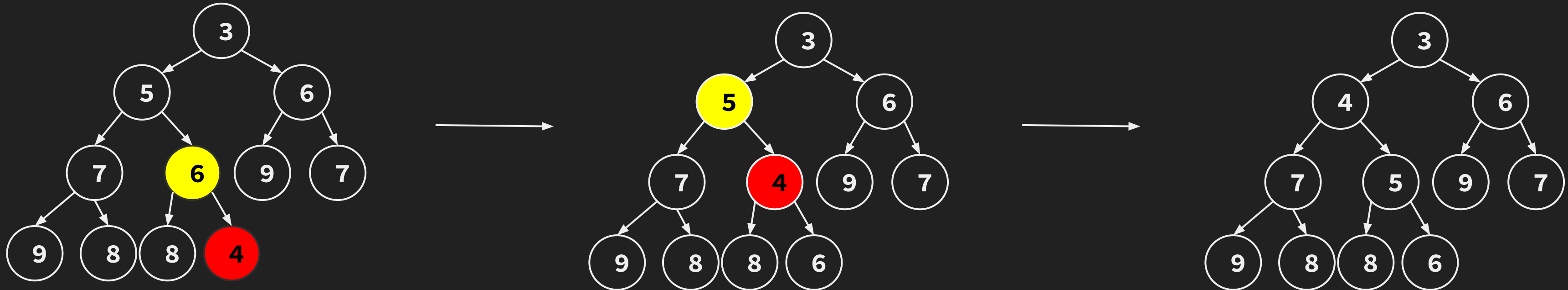
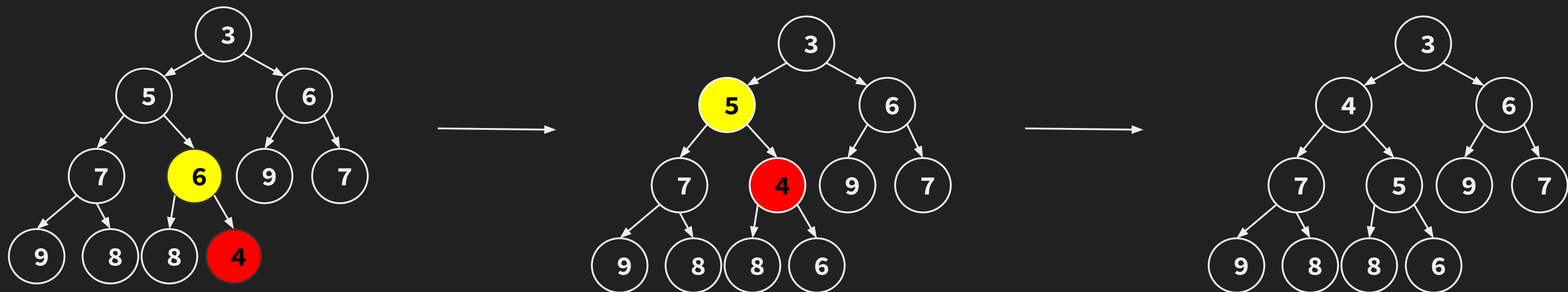
    while (node.priority is smaller than parent) {
        Swap data with parent
    }
}
```



Runtime?

“Bubble Up”

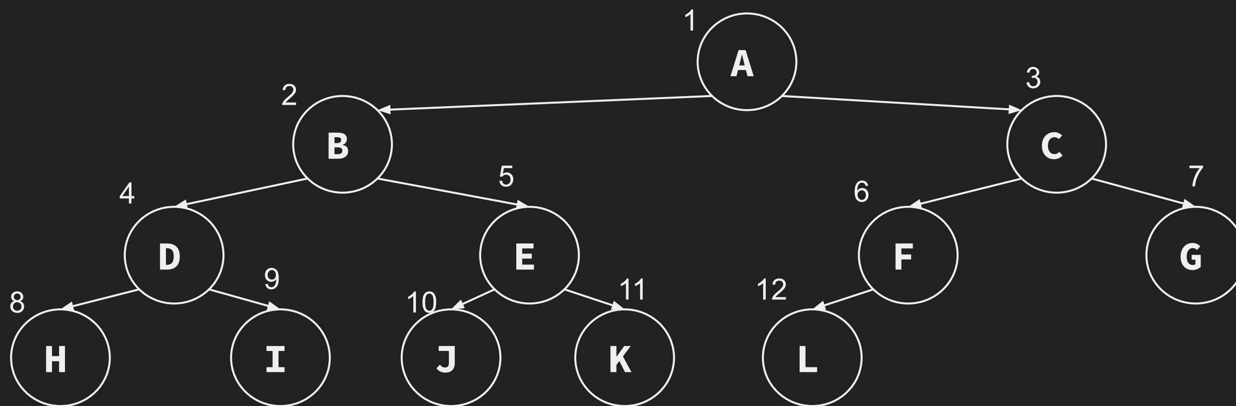
```
bubbleUp(node) {
    while (node.priority is smaller than parent) {
        Swap data with parent
    }
}
```



Runtime? Worst case, swap through every layer. Heap height is ~lg n, so runtime is O(lg n).

Implementation: array

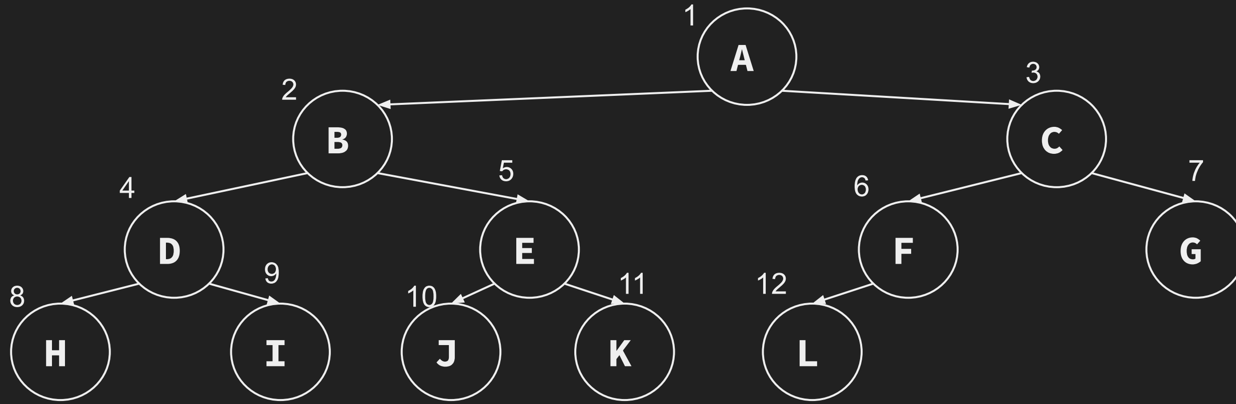
Heap: Implementation



Fill an array in **level-order** of the tree (starting from index 1):

heap:	h[0]	h[1]	h[2]	h[3]	h[4]	h[5]	h[6]	h[7]	h[8]	h[9]	h[10]	h[11]	h[12]	h[13]	h[14]	h[15]
	∅	A	B	C	D	E	F	G	H	I	J	K	L	∅	∅	∅

Heap: Implementation



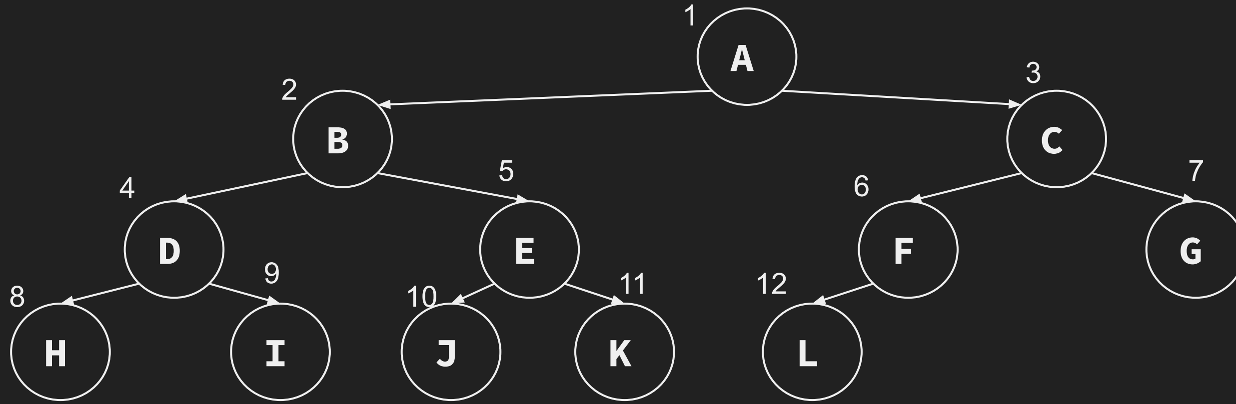
Fill an array in **level-order** of the tree (starting from index 1):

heap:	h[0]	h[1]	h[2]	h[3]	h[4]	h[5]	h[6]	h[7]	h[8]	h[9]	h[10]	h[11]	h[12]	h[13]	h[14]	h[15]
	∅	A	B	C	D	E	F	G	H	I	J	K	L	∅	∅	∅

Node at index `i` - getting its…

- Parent?
- Left child?
- Right child?

Heap: Implementation



Fill an array in **level-order** of the tree (starting from index 1):

heap:	h[0]	h[1]	h[2]	h[3]	h[4]	h[5]	h[6]	h[7]	h[8]	h[9]	h[10]	h[11]	h[12]	h[13]	h[14]	h[15]
	∅	A	B	C	D	E	F	G	H	I	J	K	L	∅	∅	∅

Node at index `i` - getting its…

- Parent? `i / 2`, rounding down
- Left child? `2i`
- Right child? `2i + 1`