NOTE: The following Internal Assessment was submitted as a part of my International Baccalaureate Standard Level Chemistry class. I received a 6 out of 7 in the class.

Cooling a Hot Cup of Water

Introduction

Every morning, I like to wake up with a cup of warm water. Coffee is quite bitter and unpleasant; the water is a nice way to ease into my day's work. However, the water cools down extraordinarily quickly. Looking around my house, I found a variety of different utensils that I could use to maintain the heat of my water; however, I wanted to find which was the best and the rates at which the water cooled based on the insulating effect of the different kinds of materials.

I decided to do some research. However, the websites that I went to all offered different information, of which very little was backed by a reason and even fewer had data that supported. With this in mind, I set out to determine the best cup to use every morning by collecting data on temperature lost, but more importantly, figuring out how to extrapolate how much heat is lost over time. The simplest way to do this was to heat up a consistent amount of water to a consistent temperature across multiple cups and then to measure the temperatures at one minute intervals.

Research Questions

There are a variety of questions that I hope this investigation will answer.

1. What is the effect of different materials on the rate of cooling of hot water?
2. What is the rate at which the average cup loses heat to the atmosphere? Is this rate constant? If not, how can it be adjusted to find a constant value to predict the rate of heat loss and amount of heat loss?

Newton's Law of Cooling

When something is being cooled down, it makes sense that the rate at which is being cooled down decreases as the liquid cools. For example, if I was to take a steaming hot cup of water and place it into the frigid winters of Siberia, it would lose a lot of heat in the first few seconds but barely any heat after a few hours. This is what Newton's Law of Cooling states – that there is a constant value that determines the rate at which the temperature changes based on the temperature of the liquid and the ambient temperature using differential equations[1] as follows:

$$\frac{dT}{dt} = -k(T - T_a)$$

[1] The term differential equations refers to an instantaneous change at any one point in time; in this case, it is relating to the instant change in temperature at any given moment in time based on both the ambient and water temperatures. More information can be found in Austin, D.; Keshet, L.; Sjerve, D. Other differential equations

This equation means that the rate of change of temperature, signified by T, is equivalent to the negative of some constant *k* multiplied by the difference in temperature between the ambient surroundings and the temperature of what is being cooled.

When the value of *k* is large, then the rate of heat loss is much greater than when the value of *k* is small; however, the rate is the only thing that changes. As time increases, the actual difference in end temperatures independent of the value of *k* decreases as they both will slow down heat loss until the temperature is eventually equal with the ambient surroundings.

This experiment will determine the *k* values and delve into which materials produce the lowest *k* values and are thus the best insulators of heat energy by using a cup of hot water.

Experimental Variables

The independent variable that I will be changing is the type of cup being used in each trial. There are six different materials being used for these cups and multiple trials as well. I used two – due to the time it takes to complete just one trial for a cup, I did the second trial to make sure the data from the first one was accurate.

The dependent variable is the rate at which the water cools. This will be measured using a thermometer to record the decreasing temperature every minute to be able to see the changes over time and be able to record how the changing rate at which the temperature drops.

The controlled variables are everything else in the experiment. This includes the amount of water (50 g), the thermometers, the initial temperature of the water (50°C), the ambient temperature of the room (21.6°C), the stopwatches, and the hot plate (set at high).

Materials

Stopwatches (3)	Stainless Steel Cup	Ceramic Mug	Small Glass Beaker (100 mL)
Styrofoam Cup	Paper Cup (12 oz)	Plastic Cup (7 oz)	Deionized Water (≈600 mL)
Thermometers (3)	Graduated Cylinders (3, 50 mL)	Gloves	Hot Plate
Large Glass Beaker (500 mL)			

Pictures of each material can be found at this end of this paper in Appendix A[2].

Safety

The only chemical used is water, which does not pose significant safety issues. It is considered nonhazardous (GHS classifications for Hazard Communication Standard), although it must be used carefully and as a chemical. It must not be consumed.

[2] Specifically, the cups have pictures in Appendix A as there can be slight variations in different kinds of ceramic mugs, plastic cups, etc.

The hot plate is the main caution in this lab. Make sure that all hair is tied up and kept away from the hot surface and that care is taken to avoid contact with the hot surface. Prevent any water from spilling onto the surface of the hot plate as well.

Additionally, it is important to be careful with handling the materials in this lab. Each of the beakers/cups is somewhat fragile and susceptible to being shattered, such as the glass beaker or ceramic mug, and the breaking of the thermometer may be concerning based on what it uses. Try to avoid using mercury thermometers, although if there is no alternative, special precaution to be careful is necessary.

Experimental Assumptions

There are a few assumptions that I made in this experiment. For one, I chose to ignore the actual sizes and dimensions of the cups themselves. As displayed in Appendix A, some cups had varying thickness and open-air amounts that I chose to ignore as they were not of the utmost importance regarding the actual k value itself. Additionally, I did not include the specific heats of the materials or the masses of the cups as they were not relevant to Newton's Law of Cooling.

Method

1. Set up the experiment. Take safety precautions as needed, begin warming up the hot plate, ensure there is a supply of deionized water, and collect all cups to be used.
2. Heat about 400 mL of water in a large glass beaker on a hot plate turned up to the max above 60°C. This does not need to be a precise measurement or a precise temperature as this will not be poured directly into any cup.
3. Take this heated water and measure out 3 amounts of 50 mL using 3 different graduated cylinders. Then, pour each one into a different cup (so for 3 cups)[3].
4. Wait until each one drops to 50°C. This may not happen at the same time for each cup, so start the stopwatch for the cup when it hits this point. The set-up should be like the picture below:

[3] It is technically possible to collect data from more than 3 cups at a time; however, since data needs to be collected every minute precisely and accurately, it proved too difficult for me. I settled on the compromise of doing 3 cups at one time as it allowed me to collect data quickly but correctly.

5. Record the data for each cup every minute for 20 minutes in its respective data table.
6. Now repeat steps 2-4 for the other 3 cups.
7. Finally, repeat steps 2-5 one more time to get data for the next trial.
8. Find the average for each minute for each material between the two trials. Then, find the change in temperature per minute for these averages.
9. Finally, plug these values into Newton's Law of Cooling and solve for the constant for each material.

Data Analysis

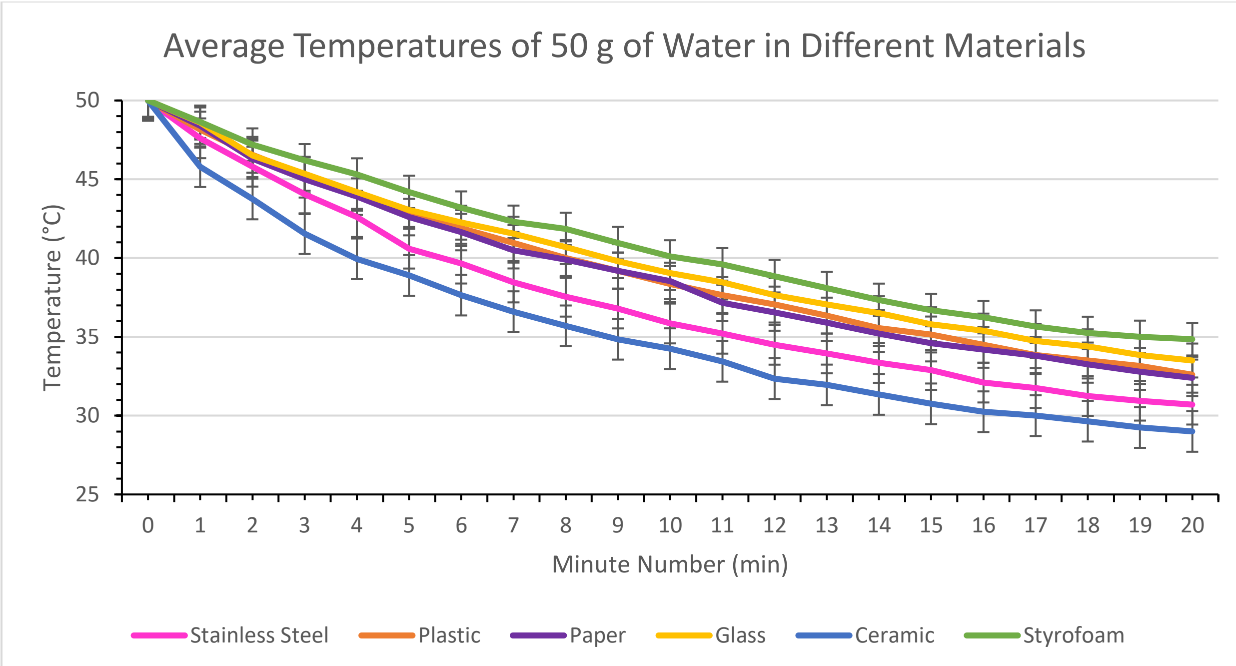
Data of Temperatures over Two Trials												
Cup:	Stainless Steel		Plastic		Paper		Glass		Ceramic		Styrofoam	
Trial:	1	2	1	2	1	2	1	2	1	2	1	2
0 min	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0
1 min	47.2	48.0	48.2	48.1	48.8	48.0	48.9	48.3	46.4	45.2	48.7	48.6
2 min	45.6	46.0	46.3	46.8	46.4	46.2	46.1	46.9	45.2	42.3	47.2	47.2
3 min	44.0	44.1	45.0	45.5	45.0	45.0	45.1	45.6	42.3	40.8	46.3	46.1
4 min	42.4	42.8	44.1	44.2	43.9	43.9	44.0	44.4	40.9	39.0	45.3	45.3
5 min	41.1	40.1	42.8	43.2	42.4	42.8	42.9	43.2	39.9	37.9	44.2	44.2
6 min	40.1	39.2	41.7	42.1	41.5	41.8	41.7	42.8	38.4	36.9	43.2	43.2
7 min	38.9	38.0	40.6	41.3	40.1	40.9	40.9	42.2	37.2	36.0	42.3	42.3
8 min	37.9	37.2	39.7	40.3	39.8	40.0	40.0	41.4	36.1	35.3	41.9	41.8
9 min	37.0	36.6	38.9	39.5	39.4	39.0	39.1	40.5	35.2	34.5	40.9	41.0
10 min	36.1	35.6	37.9	38.8	38.9	38.2	38.2	39.9	34.7	33.8	40.0	40.2
11 min	35.4	35.0	37.2	38.1	37.0	37.3	37.9	39.0	33.8	33.1	39.5	39.7
12 min	34.8	34.2	36.8	37.3	36.3	36.8	37.0	38.3	32.3	32.4	38.8	38.9

13 min	34.0	33.9	35.9	36.8	35.8	36.0	36.2	37.9	32.0	31.9	38.1	38.1
14 min	33.5	33.2	35.1	36.0	35.0	35.4	35.9	37.1	31.3	31.4	36.9	37.8
15 min	33.0	32.8	34.8	35.5	34.2	35.0	35.1	36.5	30.6	30.9	36.3	37.1
16 min	32.0	32.2	34.0	35.0	34.0	34.4	34.8	36.0	30.2	30.3	35.9	36.6
17 min	31.5	32.0	33.5	34.2	33.6	34.0	34.1	35.4	30.0	30.0	35.2	36.1
18 min	31.0	31.5	33.1	33.9	33.2	33.3	33.9	34.9	29.5	29.8	35.0	35.5
19 min	30.8	31.1	32.8	33.5	32.6	33.0	33.2	34.5	29.1	29.4	34.9	35.1
20 min	30.5	30.9	32.0	33.2	32.1	32.7	33.0	34.0	28.9	29.1	34.8	34.9

The data in the table can now be manipulated to find the average and the change in temperature per minute as well.

Average Temperatures and Changes in Temperature over Time[4]												
Cup:	Stainless Steel		Plastic		Paper		Glass		Ceramic		Styrofoam	
	Avg	$\frac{dT}{dt}$	Avg	$\frac{dT}{dt}$	Avg	$\frac{dT}{dt}$	Avg	$\frac{dT}{dt}$	Avg	$\frac{dT}{dt}$	Avg	$\frac{dT}{dt}$
0 min	50.0		50.0		50.0		50.0		50.0		50.0	
1 min	47.6	-2.4	48.15	-1.85	48.4	-1.6	48.6	-1.4	45.8	-4.2	48.65	-1.35
2 min	45.8	-1.8	46.55	-1.6	46.3	-2.1	46.5	-2.1	43.75	-2.05	47.2	-1.45
3 min	44.05	-1.75	45.25	-1.3	45	-1.3	45.35	-1.15	41.55	-2.2	46.2	-1
4 min	42.6	-1.45	44.15	-1.1	43.9	-1.1	44.2	-1.15	39.95	-1.6	45.3	-0.9
5 min	40.6	-2	43	-1.15	42.6	-1.3	43.05	-1.15	38.9	-1.05	44.2	-1.1
6 min	39.65	-0.95	41.9	-1.1	41.65	-0.95	42.25	-0.8	37.65	-1.25	43.2	-1
7 min	38.45	-1.2	40.95	-0.95	40.5	-1.15	41.55	-0.7	36.6	-1.05	42.3	-0.9
8 min	37.55	-0.9	40	-0.95	39.9	-0.6	40.7	-0.85	35.7	-0.9	41.85	-0.45
9 min	36.8	-0.75	39.2	-0.8	39.2	-0.7	39.8	-0.9	34.85	-0.85	40.95	-0.9
10 min	35.85	-0.95	38.35	-0.85	38.55	-0.65	39.05	-0.75	34.25	-0.6	40.1	-0.85
11 min	35.2	-0.65	37.65	-0.7	37.15	-1.4	38.45	-0.6	33.45	-0.8	39.6	-0.5
12 min	34.5	-0.7	37.05	-0.6	36.55	-0.6	37.65	-0.8	32.35	-1.1	38.85	-0.75
13 min	33.95	-0.55	36.35	-0.7	35.9	-0.65	37.05	-0.6	31.95	-0.4	38.1	-0.75
14 min	33.35	-0.6	35.55	-0.8	35.2	-0.7	36.5	-0.55	31.35	-0.6	37.35	-0.75
15 min	32.9	-0.45	35.15	-0.4	34.6	-0.6	35.8	-0.7	30.75	-0.6	36.7	-0.65
16 min	32.1	-0.8	34.5	-0.65	34.2	-0.4	35.4	-0.4	30.25	-0.5	36.25	-0.45
17 min	31.75	-0.35	33.85	-0.65	33.8	-0.4	34.75	-0.65	30	-0.25	35.65	-0.6
18 min	31.25	-0.5	33.5	-0.35	33.25	-0.55	34.4	-0.35	29.65	-0.35	35.25	-0.4
19 min	30.95	-0.3	33.15	-0.35	32.8	-0.45	33.85	-0.55	29.25	-0.4	35	-0.25
20 min	30.7	-0.25	32.6	-0.55	32.4	-0.4	33.5	-0.35	29	-0.25	34.85	-0.15

[4] The values in the average columns are not rounded for the sake of precision throughout the experiment, although they should be to three significant figures. This makes the *k* values calculated later overly precise, but since the values are so small, it is justified.



Clearly, the Styrofoam cup is the best heat insulator as it holds the highest temperature water at the end of the 20 minutes while the ceramic cup is the worst heat insulator as it holds the lowest temperature water at the end of the 20 minutes.

This graph also provides further proof for Newton's Law of Cooling. It is important to note that the slope is not linear for any of the lines – they all flatten out as time continues, losing the most heat at the very beginning and then conserving heat as time goes on. Therefore, the rate at which the temperature changes is based on the actual temperature itself, which is what Newton's Law of Cooling states.

From here, the *k* value can be found by using the difference between the ambient temperature (21.6°C) and the observed temperature as given by the average column for each material to determine each k value. The average of these *k* values then provides the final *k* values as well as the uncertainty in *k* values.

Manipulation of the original Newton's Law of Cooling to solve for *k* yields the following:

$$\frac{dT}{dt} = -k(T - T_a)$$

$$k = -\frac{dT}{dt} \times \frac{1}{(T - T_a)}$$

From here, the *k* values for each cup are calculated as displayed in the data table below.

k values for each cup[5]

[5] None of the k values are rounded yet for the sake of precision due to how small the k values are themselves – they are specified to the correct amount based on the uncertainties calculated in the Uncertainty section.

Cup:	Stainless Steel	Plastic	Paper	Glass	Ceramic	Styrofoam
0 min	0.092308	0.071154	0.061538	0.053846	0.161538	0.051923
1 min	0.07438	0.066116	0.086777	0.086777	0.084711	0.059917
2 min	0.077951	0.057906	0.057906	0.051225	0.097996	0.044543
3 min	0.069048	0.052381	0.052381	0.054762	0.07619	0.042857
4 min	0.105263	0.060526	0.068421	0.060526	0.055263	0.057895
5 min	0.052632	0.060942	0.052632	0.044321	0.069252	0.055402
6 min	0.071217	0.05638	0.068249	0.041543	0.062315	0.053412
7 min	0.056426	0.059561	0.037618	0.053292	0.056426	0.028213
8 min	0.049342	0.052632	0.046053	0.059211	0.055921	0.059211
9 min	0.066667	0.059649	0.045614	0.052632	0.042105	0.059649
10 min	0.047794	0.051471	0.102941	0.044118	0.058824	0.036765
11 min	0.054264	0.046512	0.046512	0.062016	0.085271	0.05814
12 min	0.044534	0.05668	0.052632	0.048583	0.032389	0.060729
13 min	0.051064	0.068085	0.059574	0.046809	0.051064	0.06383
14 min	0.039823	0.035398	0.053097	0.061947	0.053097	0.057522
15 min	0.07619	0.061905	0.038095	0.038095	0.047619	0.042857
16 min	0.034483	0.064039	0.039409	0.064039	0.024631	0.059113
17 min	0.051813	0.036269	0.056995	0.036269	0.036269	0.041451
18 min	0.032086	0.037433	0.048128	0.058824	0.042781	0.026738
19 min	0.027473	0.06044	0.043956	0.038462	0.027473	0.016484
20 min	0.092308	0.071154	0.061538	0.053846	0.161538	0.051923
Average	0.058738	0.055774	0.055926	0.052865	0.061057	0.048833

As can be seen from the above data table, the lowest *k* value is the Styrofoam cup (in green) while the highest *k* value is the ceramic cup (in red).

Additionally, qualitative observations support these findings. As stated in the First Law of Thermodynamics, energy is neither lost nor produced; it is merely transformed. The heat loss was to the atmosphere, which means that when holding these cups, the hand would feel increasingly warm with higher heat loss. When holding the ceramic cup, my hands felt quite warm compared to feeling almost nothing from the Styrofoam cup.

Uncertainty

The uncertainty for each calculation can be found by subtracting the minimum value from the maximum value and then dividing by two for the averages. Then, the same procedure will be used to calculate the uncertainty in the *k* values, with this being the goal of the investigation. The table below displays the average *k* values and the uncertainty for each. A sample calculation of the uncertainty is done below the table.

k values and uncertainty for each cup						
Cup:	Stainless Steel	Plastic	Paper	Glass	Ceramic	Styrofoam
Average	0.059	0.056	0.056	0.053	0.061	0.049

Uncertainty	0.039	0.018	0.033	0.025	0.068	0.024
% Uncertainty	66.2%	32.1%	58.4%	47.8%	112%	48.5%

Calculating the uncertainty for the stainless-steel cup is done through subtracting the maximum and minimum values of *k* and then dividing by 2. The percent uncertainty is then found by dividing this uncertainty against the average.[6]

$$k_{max} = 0.105263$$

$$k_{min} = 0.027473$$

$$k_{unc} = \frac{k_{max} - k_{min}}{2} = 0.38895 \approx 0.039$$

Results

The result of this investigation is clear in the table above. The material with the lowest *k* value and thus the best insulating material is the Styrofoam cup while the highest *k* value and the worst insulating material is the ceramic cup.

Error and Limitations

There was a significant amount of error, which was an equal combination of random due to the laboratory environment as well as systematic due to limitations I had in devising the experiment with the materials available to me.

One source is systematic - imprecise readings of the thermometers as instruments as they only displayed whole number values. A possible solution that I was considering was using the SPARK units, which would give exact computer readings of the thermometer. Unfortunately, these tended to be even more off the mark, which is why I used analog thermometers instead. Another source is random error. As expected, there was a bit of random error in natural variation as the ambient temperature may have changed slightly with different currents – other lab tables were being used for other investigations at the same time I was doing mine, perhaps releasing heat and other chemicals into the air. Finally, another source of systematic error could be the shapes of the cups themselves. They did not have the same open area percentage, which could negatively affect the results – it is quite obvious that exposing hot water to more open air will result in it losing heat to the air at a faster rate, although Newton's Law of Cooling does not account for this explicitly, although the *k* value probably changes to account for this instead. It was virtually impossible to remove this source of error by using exact similar size cups.

It was not possible to necessarily create a line of best fit relating the cup material to the *k* values, as the cups themselves do not have numerical values. I considered trying to relate each cup to the specific heat of the material used to the cup, although this had two problems. First, ceramic does not have a clear, published value for me to use – and calculating the specific heat myself would have been time-consuming and most likely with a high degree of uncertainty. Secondly, and more importantly, the other specific heat

[6] Rounding of the uncertainty is done at the very end for each calculation to 2 significant figures to maintain precision without rounding for the minimum and maximum values of the *k* values.

values that I was able to procure did not have any correlation with the *k* values that was even close to being significant.[7]

Conclusion

This investigation has yielded answers to both research questions guiding the experiment. First, different materials have a significant effect on the rate of heat loss for hot water, as evidenced by the changes in *k* values among the different cups that were used. Second, this rate is not constant but can be predicted using Newton's Law of Cooling for any type of material. Integrating the differential equations into the general form[8] as shown below after deriving the *k* values allows for prediction of final temperatures.

$$\frac{dT}{dt} = -k(T - T_a) = -ky \; where \; y = (T - T_a)$$

$$\frac{dy}{dt} = -ky$$

$$y(t) = y_0 e^{-kt}$$

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

Therefore, by solving for the *k* value, knowing the ambient temperature, and the initial temperature of the liquid, predictions over any time can be made. Of course, the *k* value changes for different types of liquids and different types of containers as well to reflect certain chemical properties and structures.

Further Investigations

A significant addition to this investigation would be to look into thermal conductivity, which looks at heat conduction to find the rate of heat transfer based on the amount of heat flow, length and surface area of the material, and temperature gradient to yield a direct value operating outside of differential equations. However, that lies beyond the scope of this investigation because of the complexity required – for the cup, there are cylindrical walls, a flat bottom area, and an open surface on the top which would be treated as its own surface. As such, that would be further investigation which would then yield its own constant that could be just as, if not more, useful in determining the usefulness of materials.

Another possible addition to the investigation is to add a lid to the top of each cup made from the same material. This is what I had planned initially – adding a ceramic plate to the top of the ceramic cup, a stainless-steel plate to the top of the stainless-steel cup, etc. However, the actual process of collecting data was very time-consuming, take me 5 hours in total of repeatedly cooling and heating the water for just 2 trials. Adding this would have effectively doubled the time it took to collect data. Since I collected the data at my school laboratory, which is susceptible to frequent temperature changes based on factors beyond my control, I collected data in one run. Having to collect data for more than 5 hours would make this impossible, further adjusting the ambient temperature and adding more sources of uncertainty to the

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[7] See Appendix B for a table of the values relating the specific heats to the *k* values.

[8] The formula and derivation of the general form of Newton's Law of Cooling is also from Austin, D.; Keshet, L.; Sjerve, D. Other differential equations.

investigation. Nonetheless, it would provide useful information about whether adding a lid is useful to the containers by comparing the new *k* values to the original ones.

References (ACS)

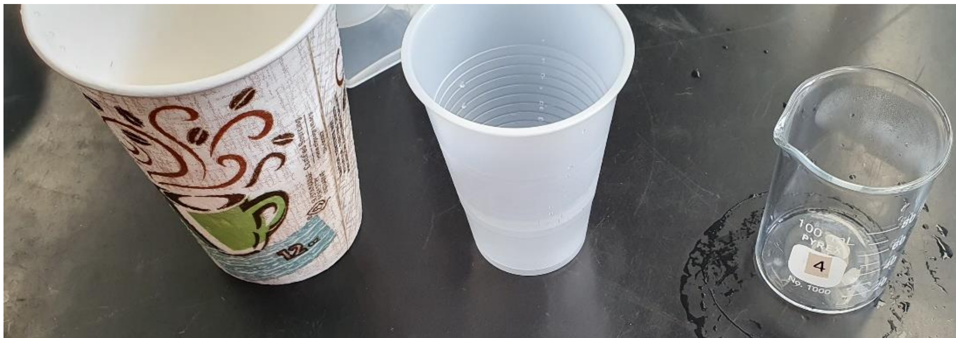
Austin, D.; Keshet, L.; Sjerve, D. Other differential equations. https://www.ugrad.math.ubc.ca/coursedoc/math100/notes/diffeqs/cool.html (accessed Nov 24, 2019).

Water Safety Data Sheet (SDS). https://www.flinnsci.com/sds_845-water/sds_845/ (accessed Nov 26, 2019).

What is Thermal Conductivity? An Overview - Thermtest. https://thermtest.com/what-is-thermal-conductivity (accessed Nov 26, 2019).

Appendix A: Cups

The following are the six cups that I chose to use in this experiment, with pictures of each.



From left to right: Paper Cup, Plastic Cup, Glass Beaker (100 mL)

From left to right: Stainless Steel Cup, Styrofoam Cup, Ceramic Cup

Appendix B: Tables

The following are additional tables of information and data that I collected in the experiment for reference.

Cup:	Stainless Steel	Plastic	Paper	Glass	Ceramic	Styrofoam
Masses (g):	55.006	39.674	338.495	49.930	316.202	1.675
Specific Heats $\left(\frac{J}{g°C}\right)$:	0.502416	1.67	1.4	0.84	?	1.131
k value / Specific Heat:	0.116911	0.033398	0.039947	0.062934	?	0.043176