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NOTE: The following Internal Assessment was submitted as a part of my International Baccalaureate Higher Level Math class. I received a 7 out of 7 in the class.

Optimizing Football Throws Based on Varying Routes

Introduction

Football is an especially fascinating sport for me because of the math involved. Beyond the statistics that many people cite regarding each of the players, looking at the math behind the throws and especially the routes has intrigued me. I never quite learned how to throw a football correctly, but I do know the ideas behind throwing the football and running the routes.

When watching Russell Wilson throw the ball at the 1-yard line against the Patriots in Super Bowl 49 instead of running the ball, I saw the interception before my eyes. Shocked, I wondered why the decision was made to throw the ball. In reality, due to the previous running plays failing and the defense setup of the Patriots at that moment, passing was the better option[1]. However, I was curious as to what the perfect throw would have looked like.

Of course, the perfect throw varies based on different kinds of routes that receivers run, coverages, and more. But I wanted to isolate just based on the receiver's route, such as the kind of play run in practice without any defensive players looking to intercept the ball to test out the play. I have decided to determine this perfect throw by using parametric equations and derivatives.

This exploration will delve into the math behind football by looking at the routes that receivers run and the optimal throwing angle and speeds that quarterbacks should aim for.

Rationale

The game of football is an American sport, so I will be using yards and seconds for my measurements throughout.

The average height of a quarterback is around six feet, three and a half inches[2]. Since every quarterback throws the ball slightly differently, I will assume that the release point of the ball is about five feet and eleven inches, or 1.97 yards, from when the quarterback throws the ball. Wide receivers, on the other hand, are about six feet and three-quarters of an inch tall[3], or 2.02 yards. While receivers can catch at a variety of different positions above and below them, using this is a good approximation of what a quarterback aims for. This also removes the need for using the small uncertainty in how quarterbacks throw the ball and other factors such as wind speed or air density since receivers can adjust to catch the ball as needed.

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[1] https://www.sbnation.com/nfl/2018/6/5/17426540/seahawks-patriots-super-bowl-49-malcolm-butler-interception-run-the-dang-ball
[2] https://www.businessinsider.com/cam-newton-is-big-2016-2
[3] https://www.footballperspective.com/average-height-of-defensive-backs-and-wide-receivers/

The speed of the football matters significantly as well. Quarterbacks, when throwing for velocity, get near 60 miles per hour[4] – for the sake of this exploration, since the point is accuracy and completions over just speed, this can be rounded down to around 50 miles per hour. In the context of the football field, converting this to yards per second is more useful for overall analysis considering the size and measurements used in the game of football.

50 miles per hour = 88,000 yards per hour ≈ 24.4 yards per second

In the 2019 NFL Combine, the fastest 40 yard dash time for any wide receiver was 4.31 seconds by Parris Campbell[5]. Considering that when wide receivers run, they are aiming for a fast time but not focusing solely on the time but rather on separation from the defensive backs, I will estimate their time to be 4.5 seconds and then use this as a value of their acceleration to map out their velocity in each of the routes based on time.

40 yards in 4.5 seconds ≈ 8.89 yards per second

The goal of this investigation is to model the path of a football and then determine the optimal throwing angle based on the type of route being run by the receiver.

Modelling the parametric equations for the football:

The football field is modeled using two variables: x to represent the forward distance, y to represent the lateral distance. Positive x will be forward and positive y will be to the right.

Using time as the variable *t* to solve for *x* and *y* on the football field using the Pythagorean Theorem and Θ for the angle from the horizontal of the ball being thrown, the equations can be written as follows from the assumptions made:

$\sqrt{x^2 + y^2} = 24.4t\cos(\Theta)$

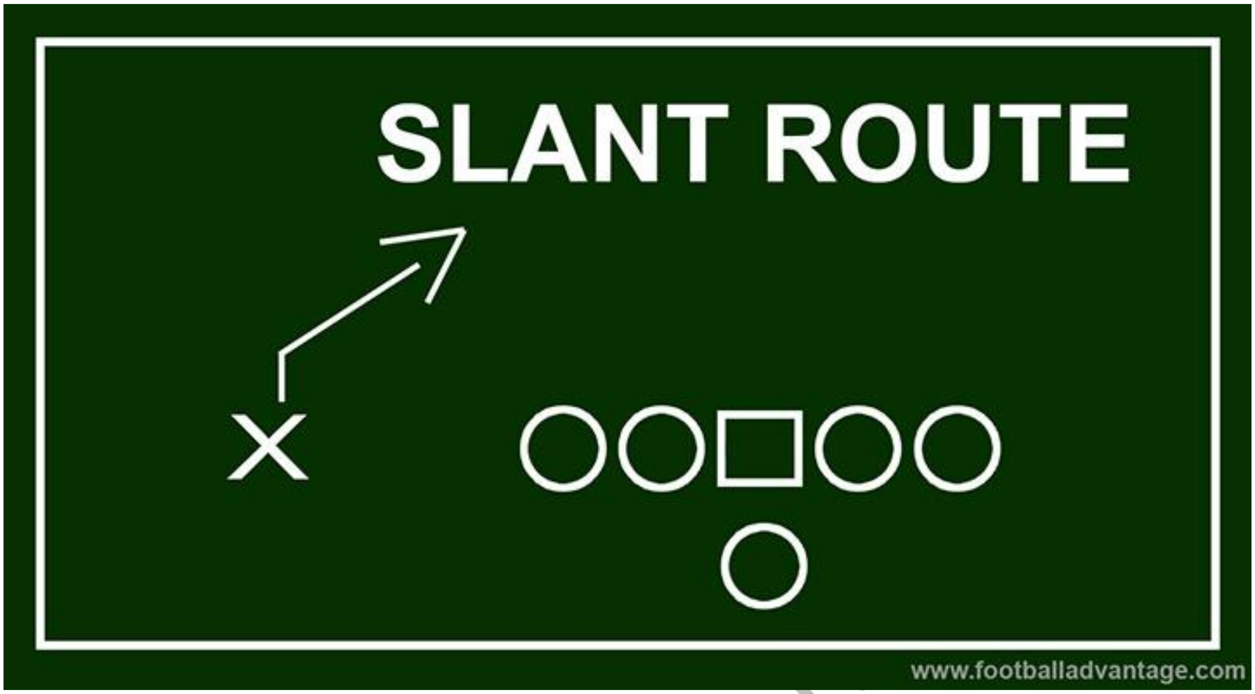
A receiver, on the other hand, can run multiple routes. The investigation analyzes their route running, of which seven common ones have been chosen. Every route assumes the receiver starts on the left side of the field, 10 yards to the left of where the ball is being snapped.

Another note for all the routes: the quarterback in every case is beginning from a "shotgun" position, meaning that he is 5 yards behind the line of scrimmage and the ball is hiked to him from the center. This simulation removes the movement of the quarterback to focus on the movement of the receivers. The quarterback is at the position where the x and y values are both 0.

Route 1: Slant

In this route, the receiver runs forward about five yards and then runs at an angle towards the open field (in this case, towards the right).

[4] https://ftw.usatoday.com/2014/03/how-fast-football-throw-nfl-combine-logan-thomas
[5] https://www.cbssports.com/nfl/draft/news/2019-nfl-combine-top-40-yard-dash-times-at-each-position-as-unheralded-safety-posts-fastest-run-at-indy/



NOTE: All football route images taken from https://footballadvantage.com/football-routes/.

The route, then, can be modeled using parametric equations. There is no z involved for any of the receivers; the receiver begins running forward and then cuts to the right. However, the quarterback does not throw to the receiver when they are running in the forward position, as the goal is to have them catch is just after cutting to the right and breaking away from the defense.

The receiver takes about 0.562 seconds to run the first 5 yards forward while being 5 yards to the left of the center and quarterback. Then, the receiver has to run at a 45 degree angle from the horizontal to the right. Since their speed while running remains at 8.89 yards per second, the x at this point is 8.89$t$cos(45°) and the y at this point is 8.89$t$sin(45°). Factoring in the initial delay and evaluating the trigonometric functions:

$$x = 6.29(t - 0.562) + 10$$

$$y = 6.29(t - 0.562) - 5$$

Plug these in to the first equation $\sqrt{x^2 + y^2} = 24.4t\cos(\Theta)$:

$$\sqrt{(6.29(t - 0.562) + 10)^2 + (6.29(t - 0.562) - 5)^2} = 24.4t\cos(\Theta)$$

$$\sqrt{(6.29t + 6.47)^2 + (6.29t - 8.53)^2} = 24.4t\cos(\Theta)$$

$$\sqrt{79.1t^2 - 25.9t + 115} = 24.4t\cos(\Theta)$$

$$\cos(\Theta) = \sqrt{79.1t^2 - 25.9t + 115} / 24.4t$$

$$-\sin(\theta)d\theta = dt\,(\sqrt{79.1t^2 - 25.9t + 115} / 24.4t)$$

Set $d\theta = 0$ to find the related rate.

$$0 = dt\left(\sqrt{79.1t^2 - 25.9t + 115} / 24.4t\right)$$

Use a graphing calculator and solve for t, which comes out to 8.88 seconds.

Thus, the optimal time to throw is 8.88 seconds into the play. Now, the angle can be found.

$$\sqrt{79.1(8.88)^2 - 25.9(8.88) + 115} = 24.4(8.88)\cos(\Theta)$$

$$\cos(\Theta) = 0.104$$

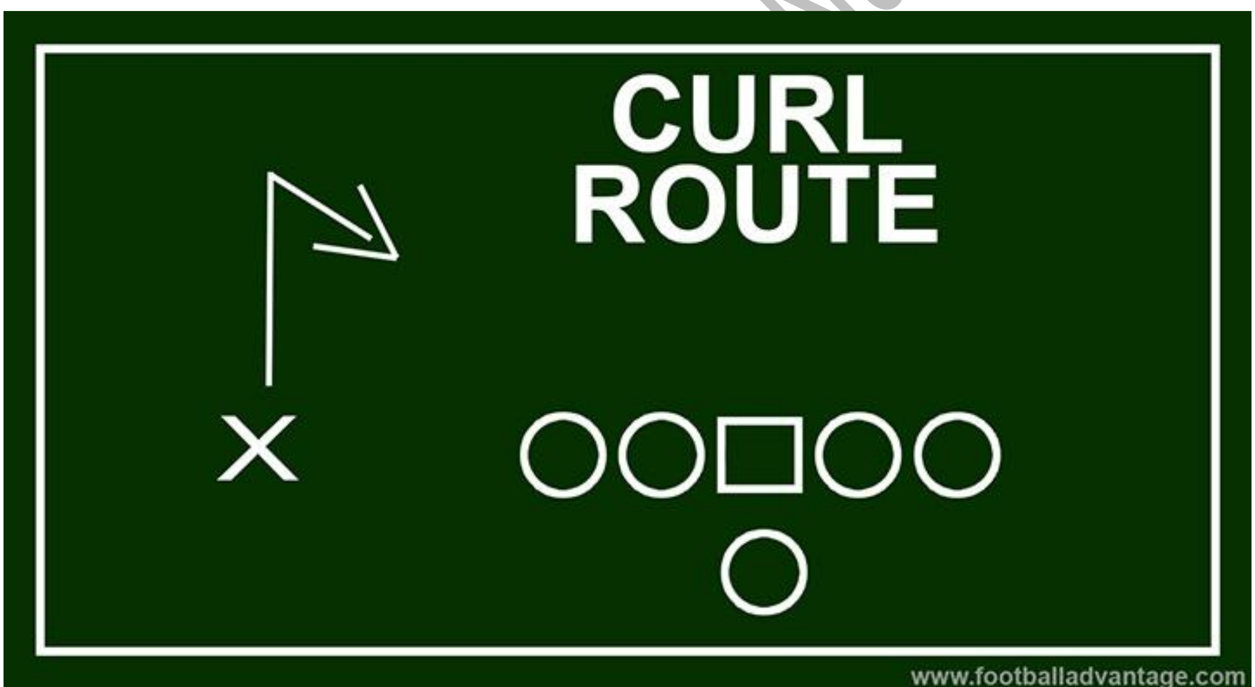
$$\Theta = 84.0°$$

Thus, the quarterback should throw it 8.88 seconds into the play at an angle of 84.0° from the horizontal to optimize the rate of the ball being thrown in relation to the receiver's route.

A similar method will be used for each of the following routes.

Route 2: Curl

In this route, the receiver runs forward about ten yards and then runs back towards the quarterback.



The x and y components will be modeled in the second half of the route, where the receiver is running back towards the quarterback. To run the first 10 yards forward requires 1.12 seconds. Then, the angle at which the receiver runs back to the quarterback at can be solved using trigonometry. The difference in y between the receiver and the quarterback is 5 and the difference in x between the receiver and quarterback is 15. Thus, the tangent of the desired angle is 5/15, so the angle is 18.4°.

$$x = -8.89(t - 1.12)\cos(18.4°) + 15 = -8.44(t - 1.12) + 15$$

$$y = 8.89(t - 1.12)\sin(18.4°) - 5 = 2.81(t - 1.12) - 5$$

Plug these in to the first equation $\sqrt{x^2 + y^2}$ = 24.4$t$cos(Θ):

$$\sqrt{(-8.44(t - 1.12) + 15)^2 + (2.81(t - 1.12) - 5)^2} = 24.4t\cos(\Theta)$$

$$\sqrt{(-8.44t + 24.5)^2 + (2.81t - 8.15)^2} = 24.4t\cos(\Theta)$$

$$\sqrt{79.1t^2 - 459t + 667} = 24.4t\cos(\Theta)$$

$$\cos(\Theta) = \sqrt{79.1t^2 - 459t + 667} \,/\, 24.4t$$

$$-\sin(\theta)d\theta = \ dt\,(\sqrt{79.1t^2 - 459t + 667} \,/\, 24.4t)$$

Set $d\theta = 0$ to find the related rate.

$$0 = dt\,(\sqrt{79.1t^2 - 459t + 667} \,/\, 24.4t)$$

Use a graphing calculator and solve for t, which comes out to 2.91 seconds.

Thus, the optimal time to throw is 2.91 seconds into the play. Now, the angle can be found.

$$\sqrt{79.1(2.91)^2 - 459(2.91) + 667} = 24.4(2.91)\cos(\Theta)$$

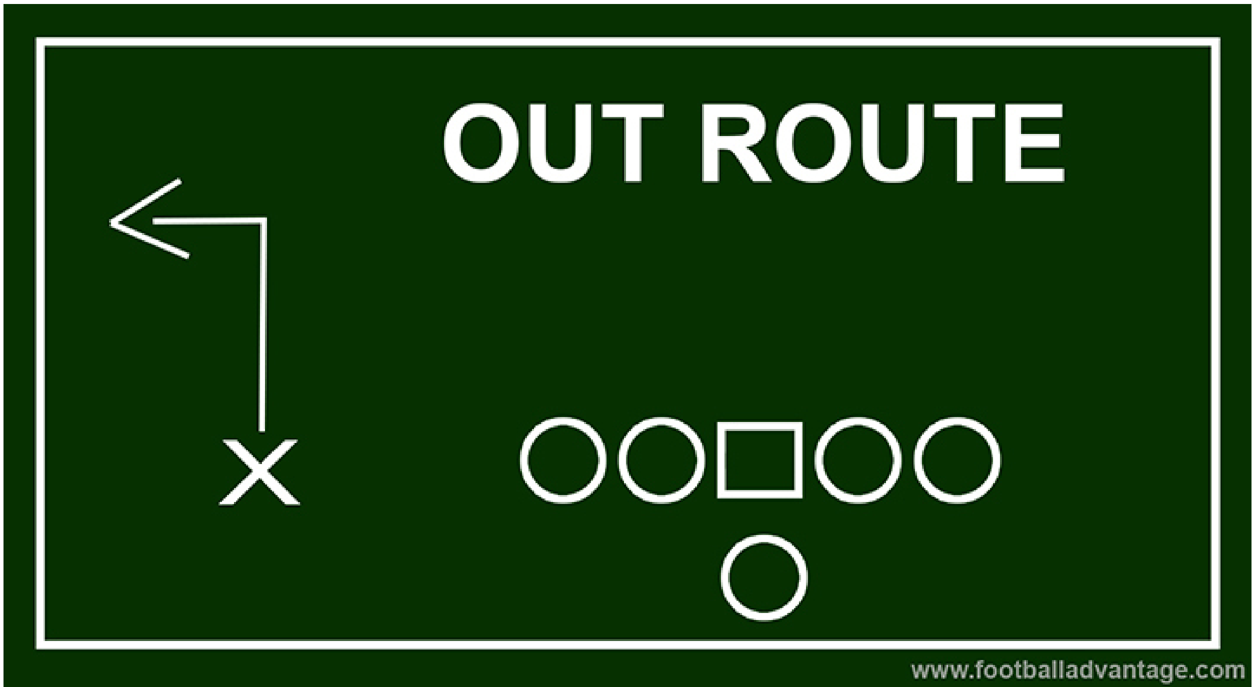
$$\cos(\Theta) = 0.0150$$

$$\Theta = 89.1°$$

Thus, the quarterback should throw it 2.91 seconds into the play at an angle of 89.1° from the horizontal to optimize the rate of the ball being thrown in relation to the receiver's route.

Route 3: Out

In this route, the receiver runs forward about ten yards and then cuts away from the quarterback towards the sidelines (in this case, towards the left).



The x and y components will be modeled in the second half of the route, where the receiver is running away from the quarterback. To run the first 10 yards forward requires 1.12 seconds. Then, the receiver has no change in the x component but runs directly to the left, changing the y component at the full speed of 8.89 yards per second.

$$x = 15$$

$$y = -8.89(t - 1.12) - 5$$

Plug these in to the first equation $\sqrt{x^2 + y^2} = 24.4t\cos(\Theta)$:

$$\sqrt{(15)^2 + (-8.89(t - 1.12) - 5)^2} = 24.4t\cos(\Theta)$$

$$\sqrt{(15)^2 + (-8.89t + 4.96)^2} = 24.4t\cos(\Theta)$$

$$\sqrt{79.0t^2 - 88.2t + 250.} = 24.4t\cos(\Theta)$$

$$\cos(\Theta) = \sqrt{79.0t^2 - 88.2t + 250.} / 24.4t$$

$$-\sin(\theta)d\theta = dt\,(\sqrt{79.0t^2 - 88.2t + 250.} / 24.4t)$$

Set $d\theta = 0$ to find the related rate.

$$0 = dt\,(\sqrt{79.0t^2 - 88.2t + 250.} / 24.4t)$$

Use a graphing calculator and solve for t, which comes out to 5.67 seconds.

Thus, the optimal time to throw is 5.67 seconds into the play. Now, the angle can be found.

$$\sqrt{79.0(5.67)^2 - 88.2(5.67) + 250.} = 24.4(5.67)\cos(\Theta)$$

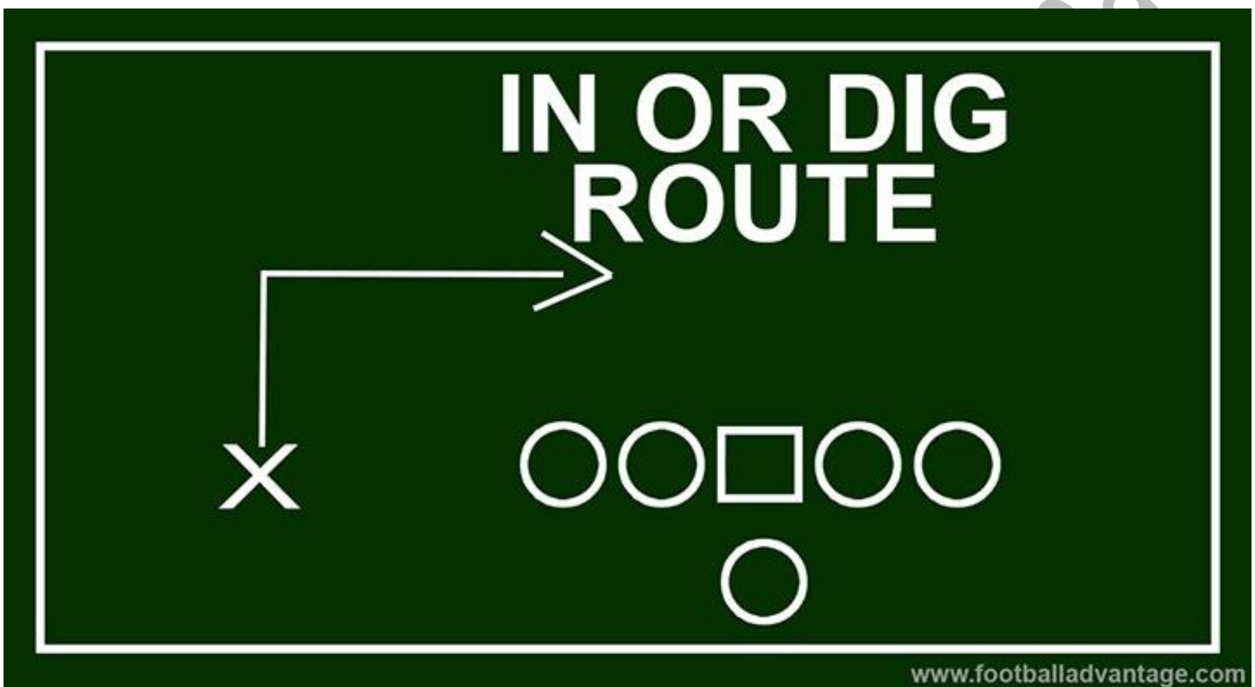
$$\cos(\Theta) = 0.346$$

$$\Theta = 69.8°$$

Thus, the quarterback should throw it 5.67 seconds into the play at an angle of 69.8° from the horizontal to optimize the rate of the ball being thrown in relation to the receiver's route.

Route 4: In/Dig

In this route, the receiver runs forward about ten yards and then cuts away from the quarterback towards the open side of the field (in this case, towards the right).



The x and y components will be modeled in the second half of the route, where the receiver is running towards the quarterback. To run the first 10 yards forward requires 1.12 seconds. Then, the receiver has no change in the x component but runs directly to the left, changing the y component at the full speed of 8.89 yards per second.

$$x = 15$$

$$y = 8.89(t - 1.12) - 5$$

Plug these in to the first equation $\sqrt{x^2 + y^2} = 24.4t\cos(\Theta)$:

$$\sqrt{(15)^2 + (8.89(t - 1.12) - 5)^2} = 24.4t\cos(\Theta)$$

$$\sqrt{(15)^2 + (8.89t - 15.0)^2} = 24.4t\cos(\Theta)$$

$$\sqrt{79.0t^2 - 267t + 450.} = 24.4t\cos(\Theta)$$

$$\cos(\Theta) = \sqrt{79.0t^2 - 267t + 450.} / 24.4t$$

$$-\sin(\theta)d\theta = \ dt\,(\sqrt{79.0t^2 - 267t + 450.}\ /\ 24.4t)$$

Set $d\theta = 0$ to find the related rate.

$$0 = dt\,(\sqrt{79.0t^2 - 267t + 450.}\ /\ 24.4t)$$

Use a graphing calculator and solve for t, which comes out to 3.37 seconds.

Thus, the optimal time to throw is 3.37 seconds into the play. Now, the angle can be found.

$$\sqrt{79.0(3.37)^2 - 267(3.37) + 450.} = 24.4(3.37)\cos(\Theta)$$

$$\cos(\Theta) = 0.257$$

$$\Theta = 75.1°$$

Thus, the quarterback should throw it 3.37 seconds into the play at an angle of 75.1° from the horizontal to optimize the rate of the ball being thrown in relation to the receiver's route.

Conclusion

There were 4 routes that I chose to analyze and model: the slant, curl, out, and in routes. The results for the optimal angles were 84.0, 89.1, 69.8, and 75.1 with times of 8.88, 2.91, 5.67, and 3.37, respectively. The optimal angles were all leading the receiver, meaning that they were not thrown at low angles like bullet passes. The times did have significant variance, from 2.91 seconds all the way up to 8.88 seconds, which could be difficult to even throw considering pressure from defensive lineman.

Of course, there were a lot of things that were taken for granted, such as the quarterback not moving in the pocket to throw, receivers following exactly perfect lines for routes without adjustment, and more. However, the overall aim of this exploration was accomplished in exploring the varying routes that receivers ran.

Reflecting on the exploration, there are a couple of things that I would have liked to incorporate that probably make my attempts at modelling not fully adequate.

For one, many quarterbacks must play differently based on their age, offensive line, and receivers. For example, a weak offensive line forces the quarterback to throw on the run, decreasing accuracy for the sake of time, while having fast receivers that are able to separate more easily from defensive backs allows for the quarterback to lead them in easier while throwing for more yardage.

As mentioned above, all of my angles were higher than would be the case for bullet passes which are thrown almost horizontally to get the ball to the receiver as quickly as possible for a few brief moments free from coverage. That may require using a different model that splits the parametric equations to account for it.

The numbers that I chose such as having the quarterback only throw from the shotgun position greatly affected some of the results. For the first route, I initially forgot to factor in the 5 yard difference of the quarterback from the line of scrimmage and got a time of 1.69 seconds, vastly different from the 8.88 seconds although the angle did not change much by less than 10 degrees. That means that many of the numbers I chose were just approximations that could alter the numerical values significantly even if slightly adjusted, reducing the applicability of the results.

Some routes did not lend themselves to being modelled by this simulation which leads me to believe that there may be some inconsistency with how I chose to find the optimal angles and times for throwing. For example, when doing a post route which is where the receiver runs out 10 yards and then cuts, I was not able to find an optimal time when taking the derivative as the derivative remained negative for over 60 seconds after 0, by which time the receiver would have been off the field. In fact, I had original plans to find the optimal angles for throws for the deeper routes like the fly route, where the receiver runs in a straight line from the line of scrimmage, but the values were simply non-existent to make it work like with the shorter cut routes.

Initially, I wanted to implement the dropback time for a quarterback which I found to be 2.75 seconds when taking the average of every quarterback's time to throw the ball after the play has begun from the 2018 NFL regular season. Unfortunately, when I then graphed the function, I got a negative value of t which was impossible since t must be a positive number as it is in terms of time. As a result, I was unable to include this time, which probably makes the investigation slightly flawed.

Additionally, this simulation ignores the influence of the defense and other wide receivers, with defensive tackles forcing quick decisions by the quarterback and defensive backs altering paths of the wide receiver to not be as ideal as the routes presented (of course, the numbers are approximates and are not always followed).

As a whole, I enjoyed doing this exploration into football. While the statistics behind football are very interesting, looking into the motion of the football and attempting to optimize it using derivatives was a complicated task that does not model the football perfectly but is definitely quite insightful for the exploration.